

Elevation-dependent vertical bias in se27_redohx_sep2026

Validation against the 2011 and 2012 Öræfajökull airborne lidar DEMs

Joaquin M. Belart · xDEM 0.2.2, geoutils 0.2.5 · 28 September 2026

The data

- **Tested DEM** — se27_redohx_sep2026_sg-DEM.tif, EPSG:8088 (ISN2016 / Lambert 2016), 1 m.
- **Reference A** — Orafajokull2_20110826_zmae.tif, 2011 airborne lidar, vertical accuracy better than 0.5 m. EPSG:3057, 2 m. Overlap 62–1812 m a.s.l., 56.3 M off-glacier pixels.
- **Reference B** — Orafajokull_20120713_zmae.tif, 2012 airborne lidar over the low sandur. EPSG:3057, 2 m. Overlap 62–938 m a.s.l., 108.6 M off-glacier pixels.
- **Glacier mask** — glacier_polygons_iceland_2007_2012.gpkg. Glaciated terrain is set to no-data before any statistic: 47.3 % of the 2011 overlap, 1.5 % of the 2012 one. Glacier change plays no part in any number here.
- **Geoid** — Icegeoid_ISN2016.gtx.
- **Convention** — dh = reference – se27, so negative dh means se27 sits *above* the lidar. Each pair is resampled onto a common 2 m grid in EPSG:3057.

The tests

- Raw difference on the common grid, no co-registration.
- Nuth & Kaab co-registration, fitted on off-glacier terrain (slope 0.1–40°, |dh| under 100 m).
- dh binned into 25 m elevation bands of the reference; a straight fit through the binned medians gives the bias slope in m per 100 m.
- NuthKaab + TerrainBias with terrain_attribute = "elevation", validated first on synthetic terrain carrying a known –0.10 m per 100 m bias.
- All of the above repeated against a second, independent lidar reference covering a different footprint.
- Geoid undulation over both areas: its range, and its regression against terrain elevation.

The findings

- **se27 is stretched vertically** — too high at low ground, too low on the upper slopes. The bias is proportional to elevation at **+0.117 m per 100 m** before co-registration.
- **Co-registration cannot remove it.** Nuth & Kaab (dx –0.409, dy –0.473, dz –1.237 m) moves the median from –1.184 to +0.021 m, but the slope stays at +0.102 m per 100 m and NMAD at 0.497 m. A rigid transform has no height-dependent degree of freedom.
- **TerrainBias against elevation removes it.** Slope falls to +0.003 m per 100 m, NMAD halves to 0.248 m, and the profile is flat to within ±0.03 m from 62 m to 1562 m.
- **Independently confirmed.** The 2012 lidar, over a different and lower footprint, gives +0.088 raw, +0.083 after NuthKaab and –0.001 after TerrainBias, in m per 100 m.
- **Magnitude** — about 2 m across 1750 m of relief, a vertical scale error of roughly 0.11 %. Consistent with a slightly miscalculated focal length in a nadir photogrammetric block.
- **Geoid or ellipsoid is not the cause.** N is 65–67 m here, so a frame mix-up would appear as a ~66 m offset; the observed medians are –1.18 and –1.78 m. N does rise with elevation (+0.060 and +0.051 m per 100 m), so such an error would not have been a pure shift — but that is half the measured slope, with no mechanism to produce it.
- **After correction** the agreement with the lidar, 0.248 m NMAD, is inside the lidar’s own 0.5 m vertical accuracy.

	Median (m)	NMAD (m)	Slope (m/100 m)	Profile spread (m)
Reference A — 2011 lidar, 62–1812 m a.s.l., n = 56,280,733 px				
No co-registration	–1.184	0.543	+0.117	2.379
Nuth & Kaab	+0.021	0.497	+0.102	1.986
Nuth & Kaab + TerrainBias	+0.000	0.248	+0.003	0.489
Reference B — 2012 lidar, 62–938 m a.s.l., n = 108,625,436 px				
No co-registration	–1.780	0.256	+0.088	0.900
Nuth & Kaab	+0.001	0.242	+0.083	0.832
Nuth & Kaab + TerrainBias	–0.001	0.167	–0.001	0.025

Full off-glacier data, no outlier mask. Slope is a straight fit through the binned medians; profile spread is peak-to-peak across them. The outlier mask ($|dh - \text{median}| > 3 \times \text{NMAD}$) used in the maps overleaf is applied to the maps only — histograms show the full distribution with the rejection window drawn on them, and the elevation profiles use unclipped data, because clipping about a global median would cut high-elevation pixels hardest and flatten the very trend being measured.

Elevation-dependent bias: $dh = 2011 \text{ lidar} - \text{se27_redohx_sep2026}$

Reference: 2011 airborne lidar, vertical accuracy better than 0.5 m. Glaciers masked out.

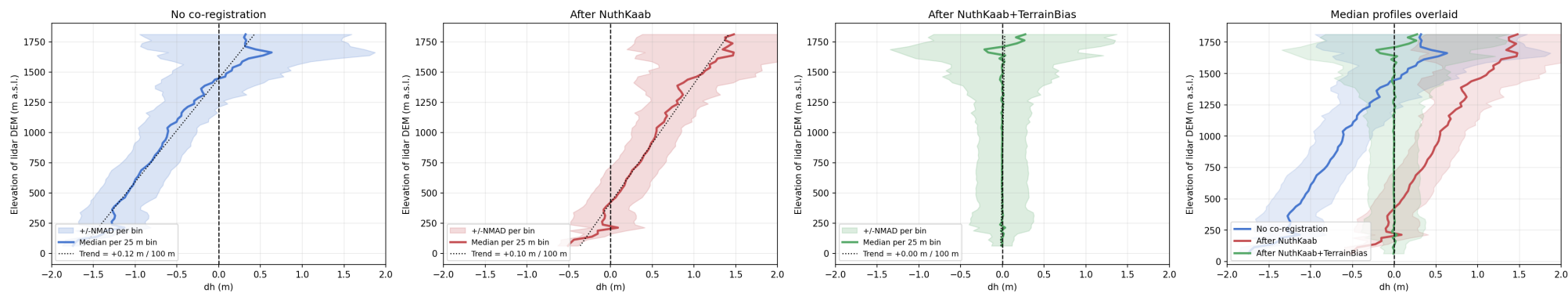


Figure 1. The bias, and why co-registration misses it (reference A, 2011 lidar).

dh against lidar elevation. Grey hexbins are pixel density on a log count scale; the coloured line is the median per 25 m band with its $\pm$ NMAD envelope; the dotted line is a straight fit through the binned medians. The first two profiles are near-parallel — Nuth & Kaab translated the curve by its 1.237 m vertical shift without changing its slope. The third, after TerrainBias, is vertical.

dh = 2011 lidar - se27_redohx_sep2026 (after NuthKaab)

Reference: 2011 airborne lidar, vertical accuracy better than 0.5 m. Glaciers masked out.

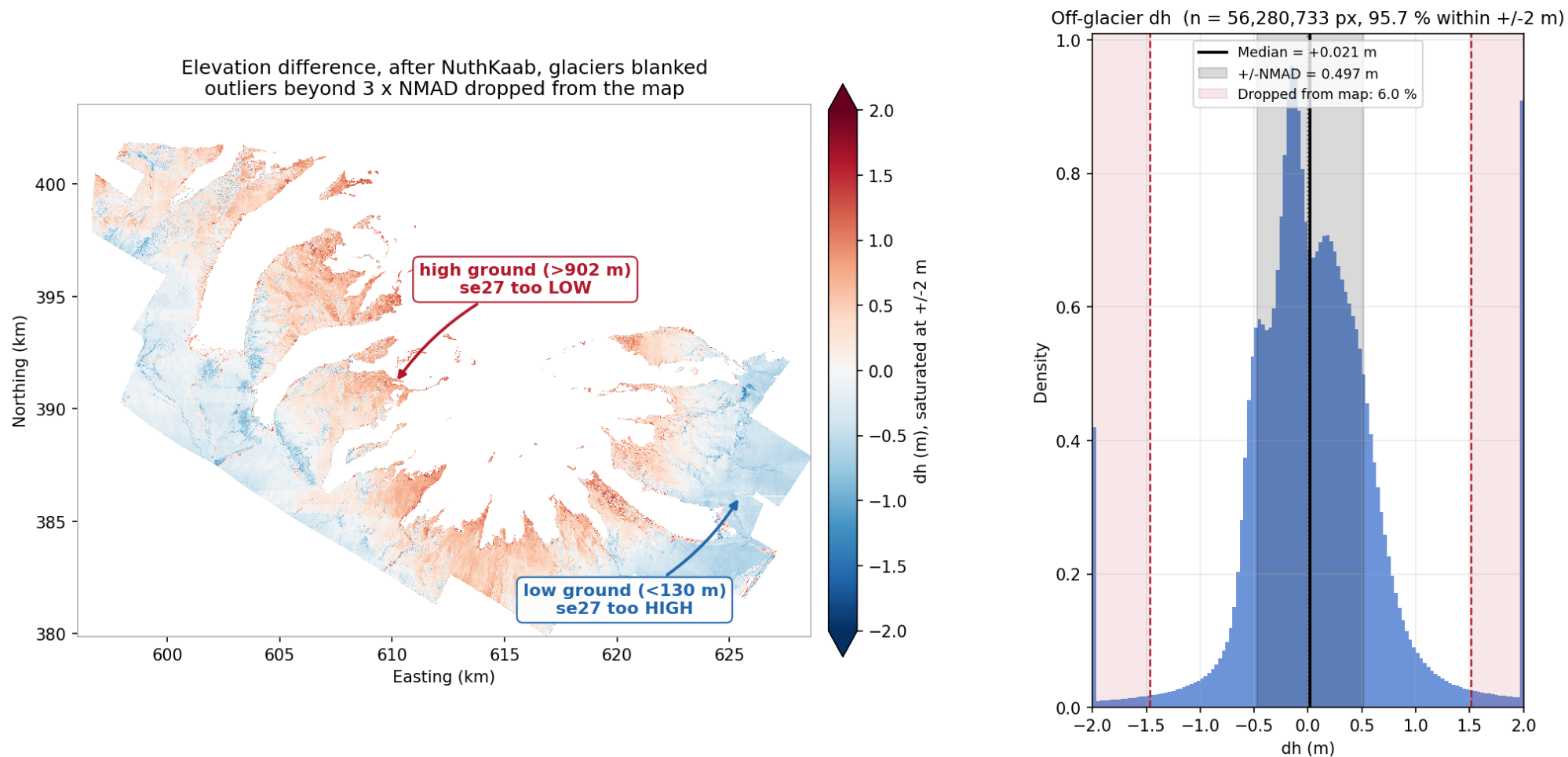


Figure 2. Where the bias lands on the ground (reference A, after Nuth & Kaab).

Residual after co-registration, ± 2 m scale, glaciers blanked. The low outwash plain is blue, the high flanks ringing the ice cap are red: the pattern follows elevation, not flight lines or scene edges. Arrows mark the median location of the lowest and highest 12 % of off-glacier terrain. Median +0.021 m, NMAD 0.497 m over 56,280,733 px. The histogram shows the full distribution; the pink bands are the $3 \times$ NMAD window dropped from the map only.

dh = 2011 lidar - se27_redohx_sep2026 (after NuthKaab-TerrainBias)

Reference: 2011 airborne lidar, vertical accuracy better than 0.5 m. Glaciers masked out.

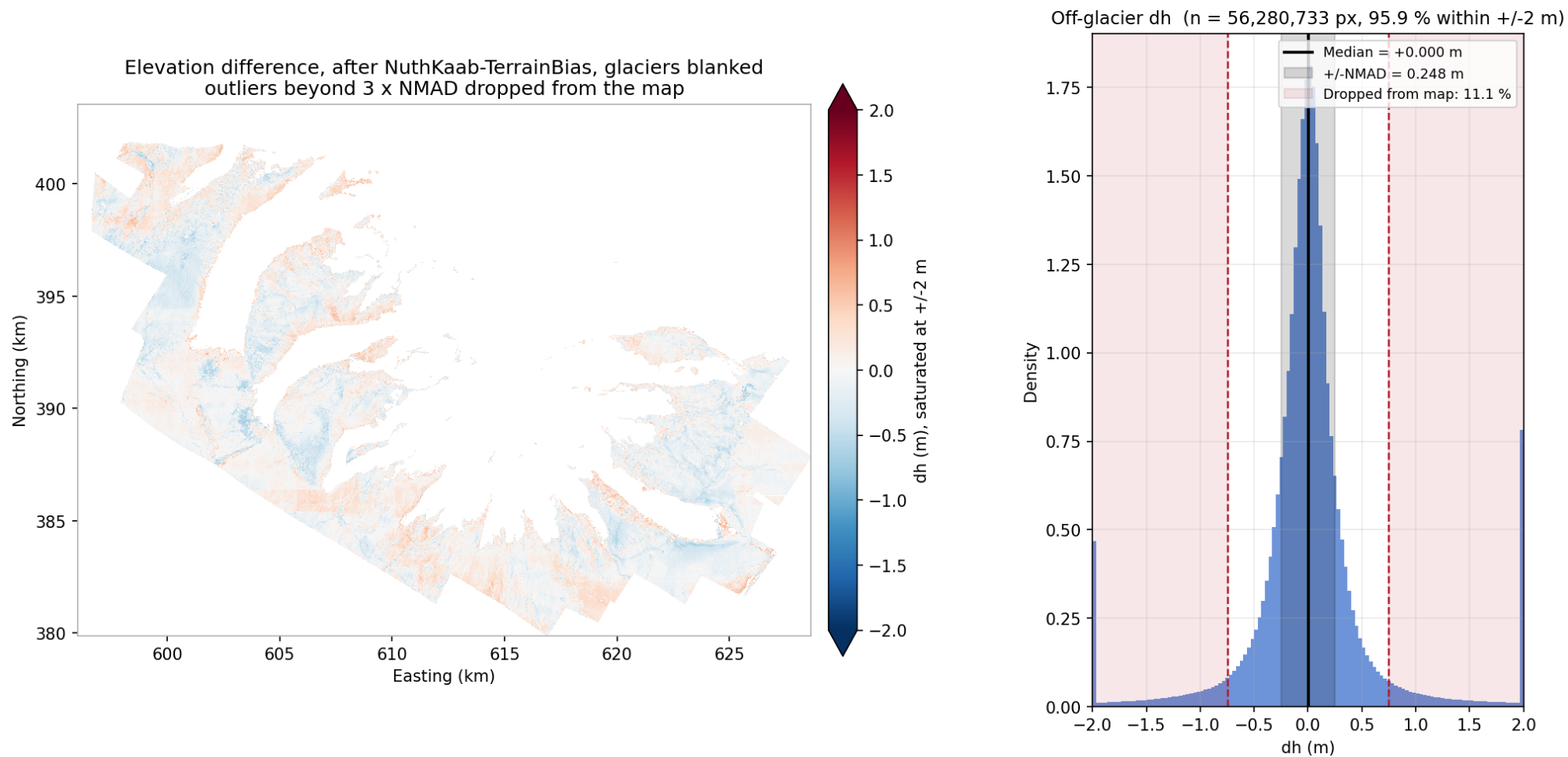


Figure 3. After NuthKaab + TerrainBias against elevation (reference A).

Same ± 2 m scale. The high-and-low pattern of Figure 2 is gone and the map is essentially featureless. Median +0.000 m, NMAD 0.248 m — half the value before the correction, which says the elevation bias was the dominant residual error in the scene rather than a tilt on top of noise.

Elevation-dependent bias: dh = 2012 lidar - se27_redohx_sep2026

Reference: 2012 airborne lidar survey. Glaciers masked out. Overlap spans 62-938 m a.s.l.

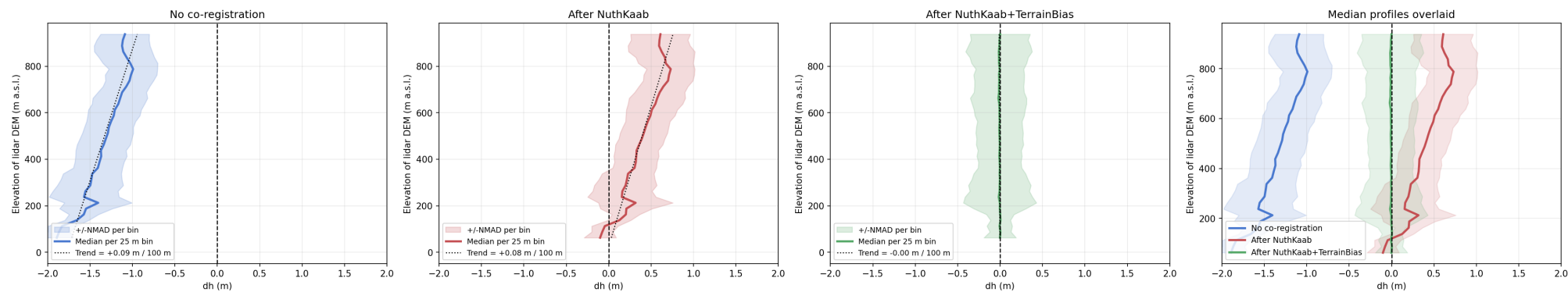


Figure 4. Independent confirmation (reference B, 2012 lidar).

dh against lidar elevation over the 2012 pair's narrower 62–938 m range, covering a different part of se27. Same sign, same magnitude, same collapse after TerrainBias: +0.088 raw, +0.083 after Nuth & Kaab, –0.001 after TerrainBias, in m per 100 m. Two lidar surveys sharing no acquisition, no epoch and only part of their footprint agree, so the bias is a property of se27.

Could a geoid / ellipsoid mix-up explain the bias?

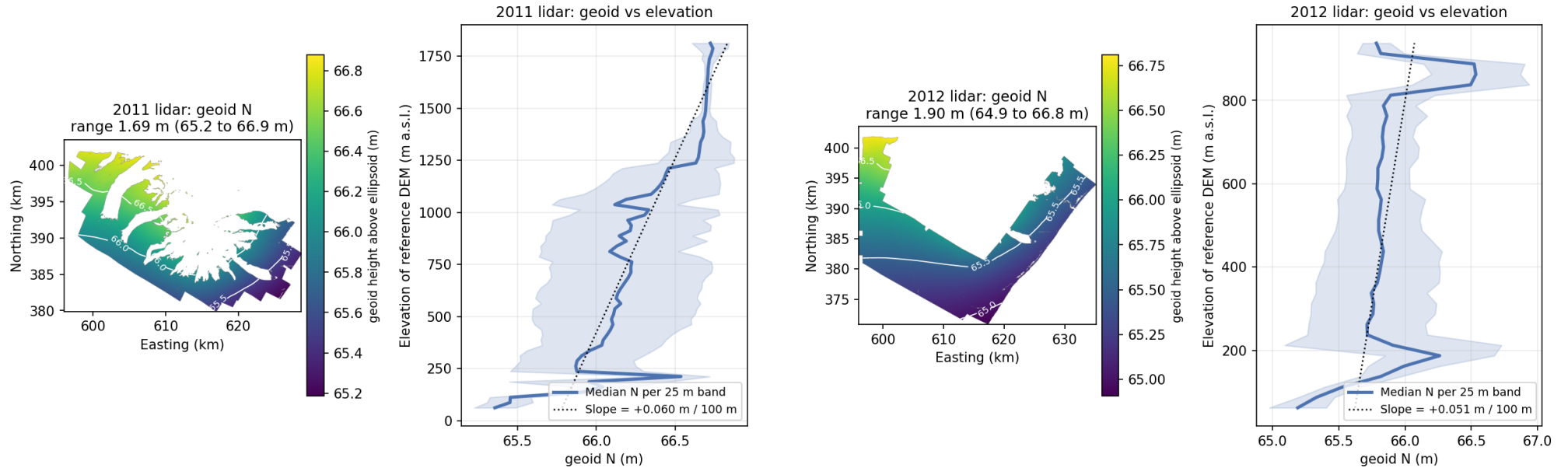


Figure 5. The geoid over both comparison areas.

Icegeoid_ISN2016 sampled on the same 2 m grids and masked to the same off-glacier pixels; white contours every 0.5 m. N is 65–67 m above the ellipsoid, so a geoid/ellipsoid mix-up would appear as a ~66 m offset, which is not present. N is not flat with elevation, rising +0.060 m per 100 m (2011) and +0.051 m per 100 m (2012), so such an error would not have been a simple vertical shift — but that is about half the measured bias slope. Because N and terrain height are collinear here, the two cannot be separated by regression; the geoid is excluded on magnitude and on the missing offset.