# Leveling adjustment and land uplift

## Calculation of a land uplift model

- **Observations**
- **The method**
- **Results**

***Reykjavík***

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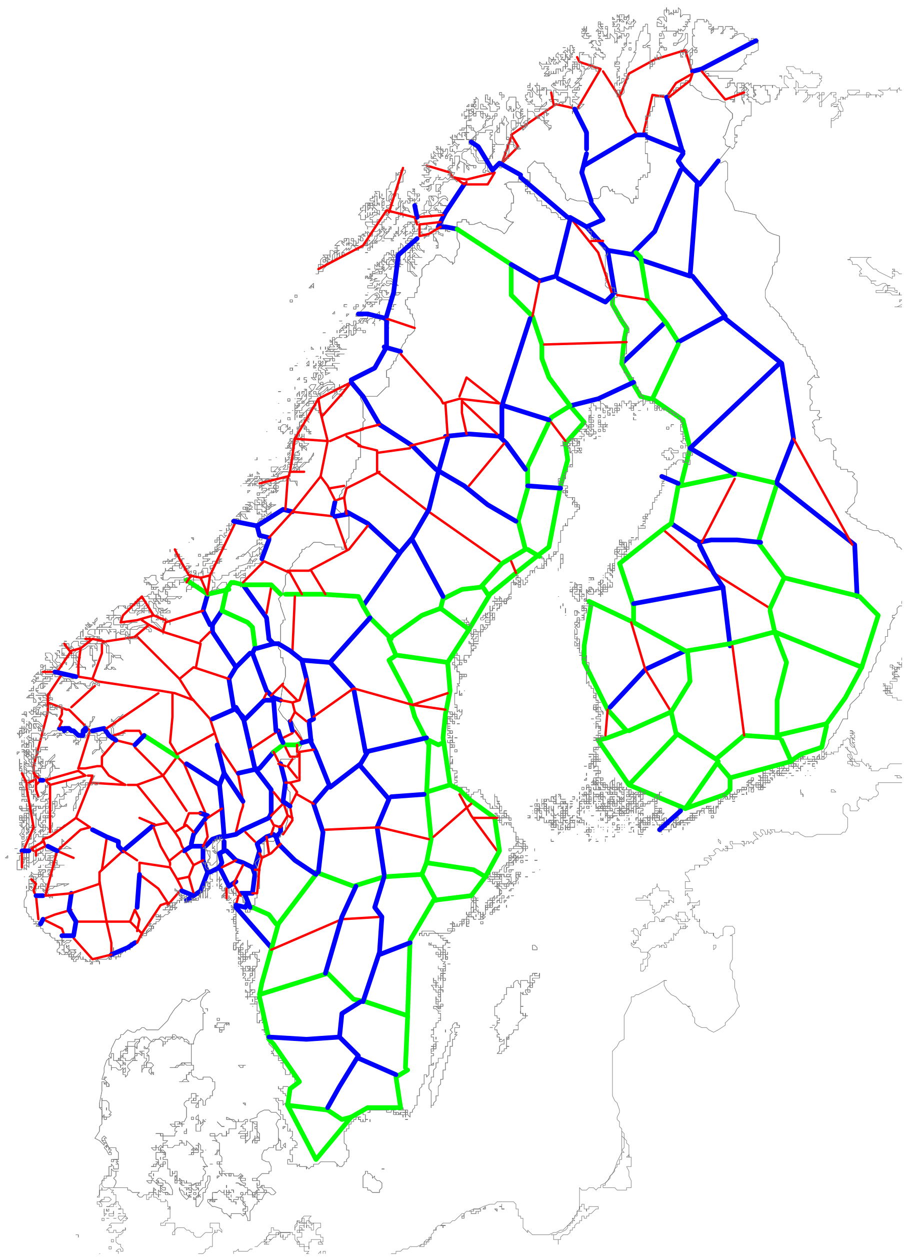
# Observations

- **Leveling observations**
  - ***From Norway, Sweden and Finland***
  - ***Geopotential differences between nodal points***
  - ***From 1890 - 2003***
- **Tide gauges**
  - ***58 stations***
  - ***Martin Ekman's values, published in 1996.***
- **GPS-rates**
  - ***55 stations in BIFROST***
  - ***Results from Martin Lidberg's Licentiate Thesis***

# *Leveling network*



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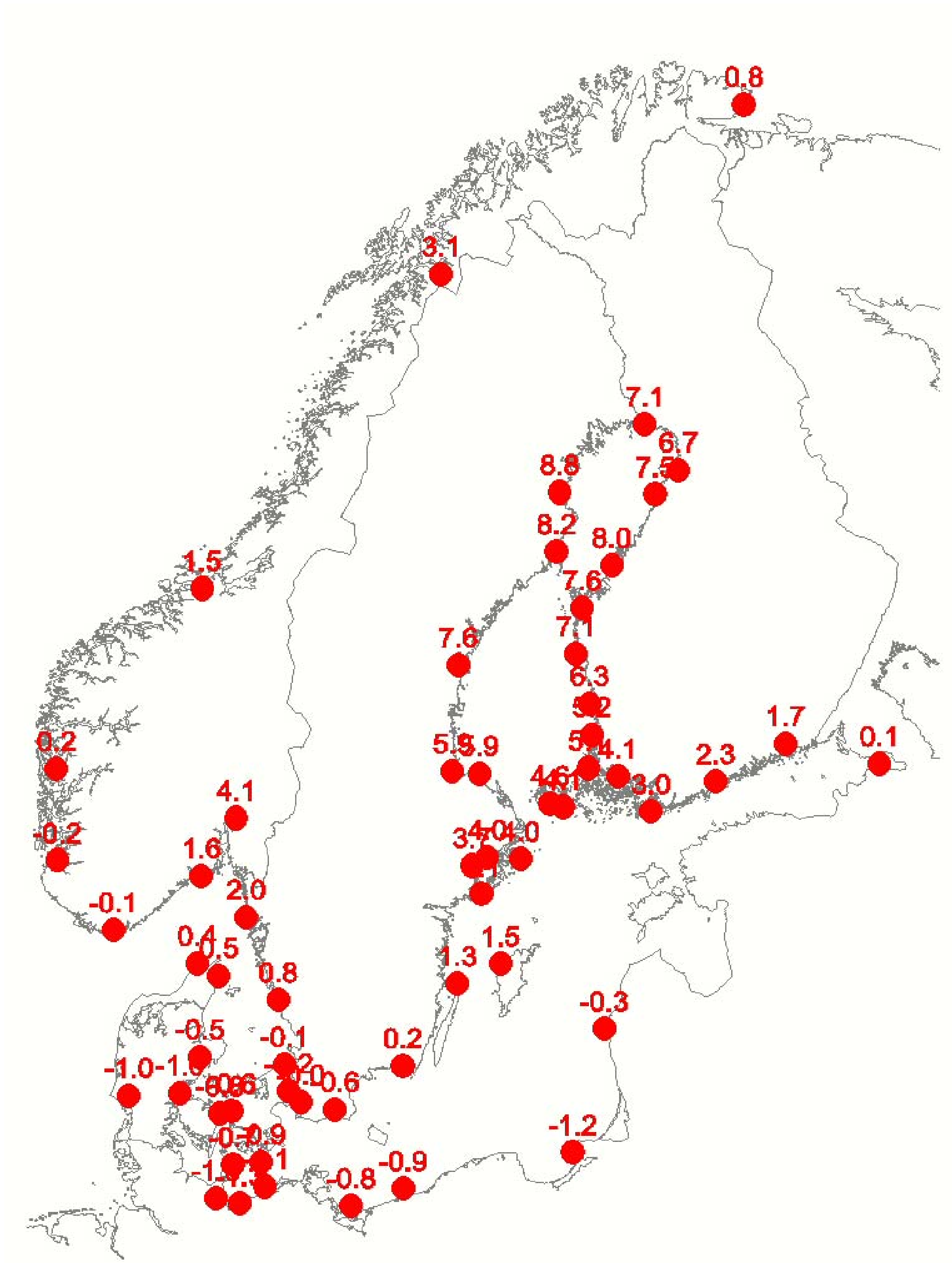
# Tide gauges

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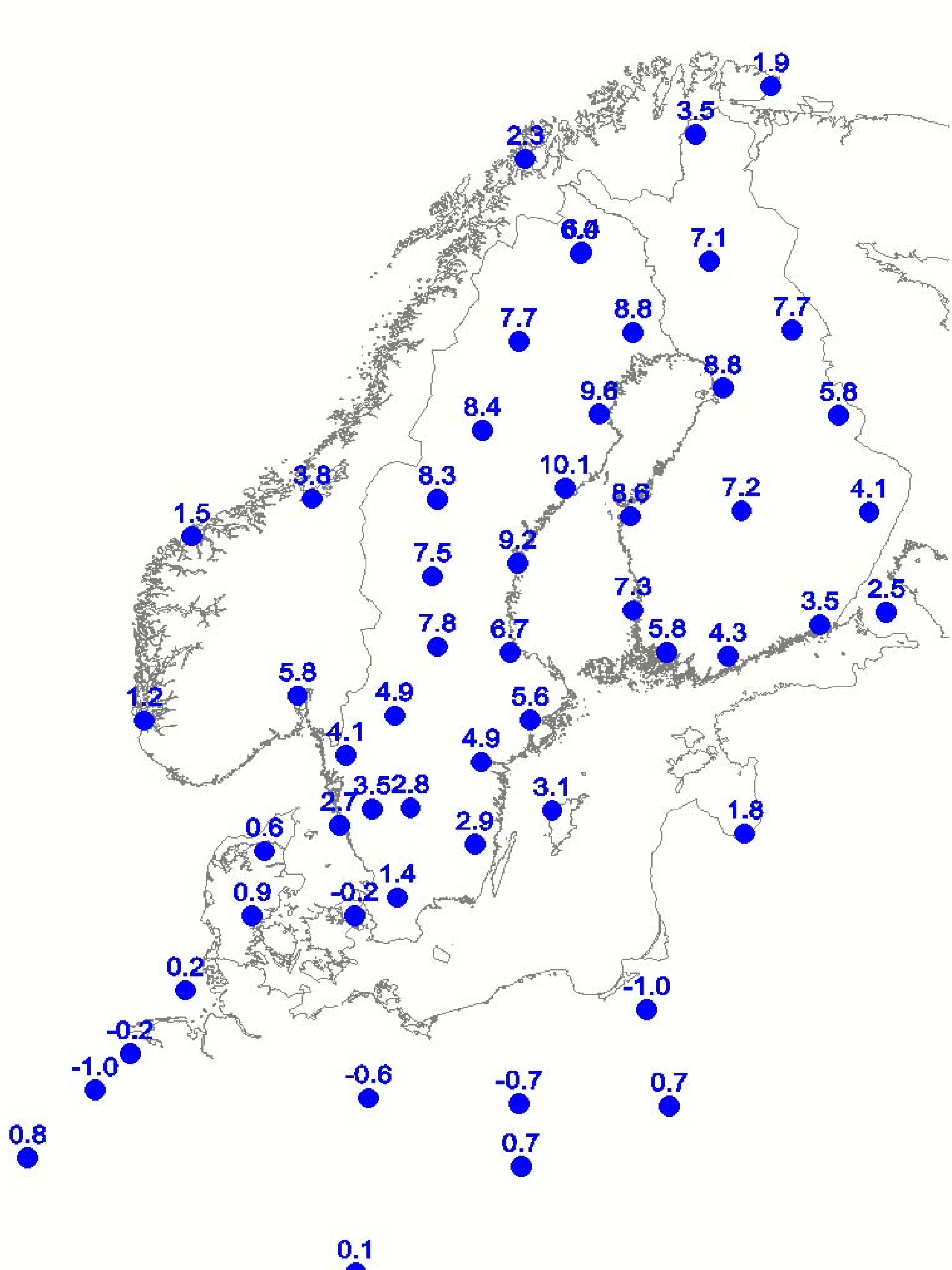
# Permanent GPS-stations

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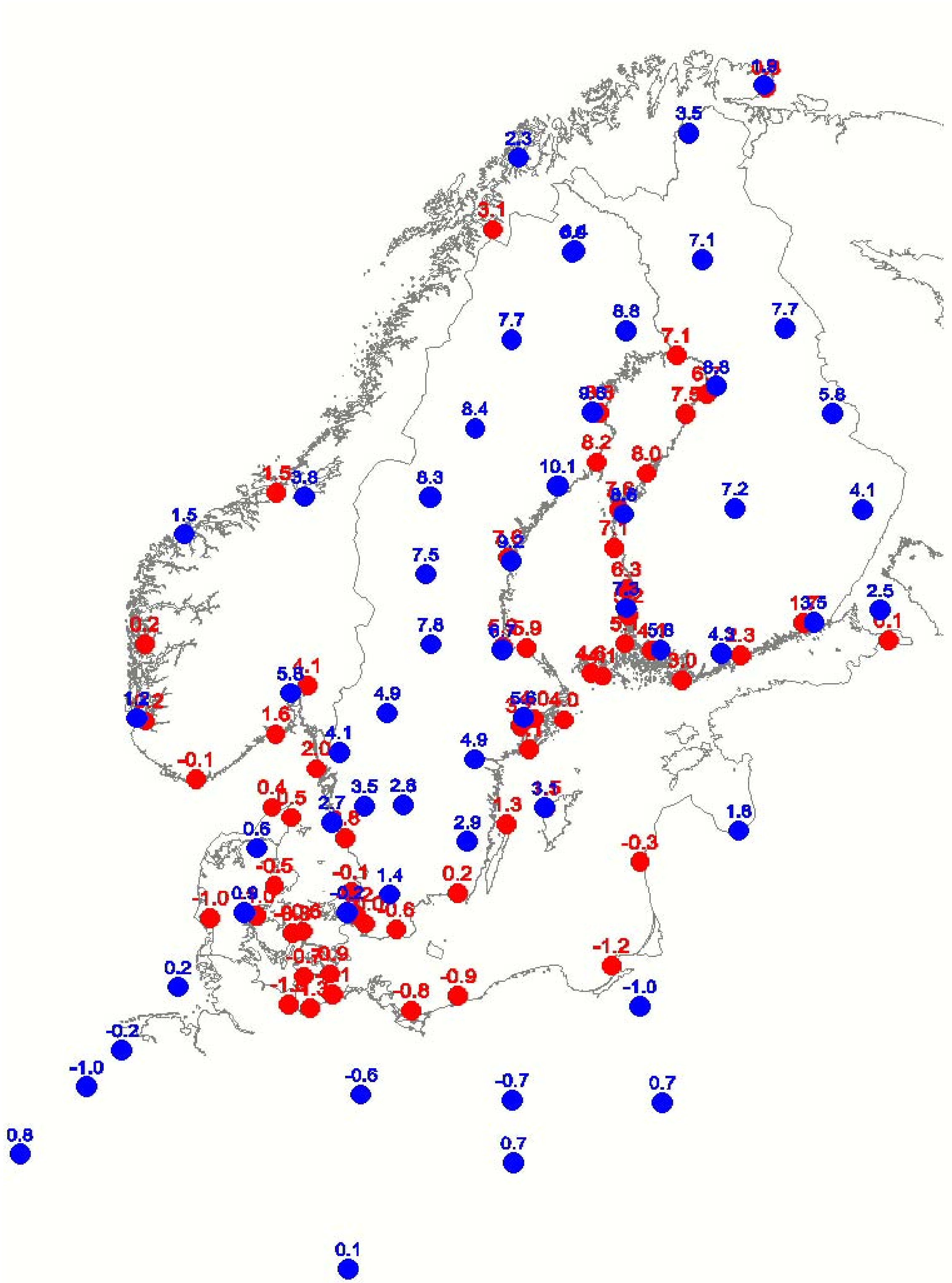
# GPS and tide gauges



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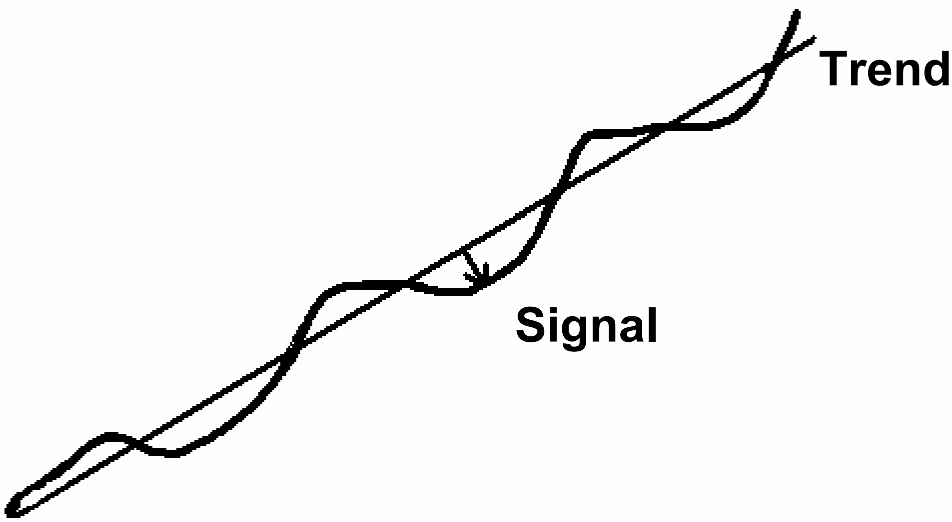
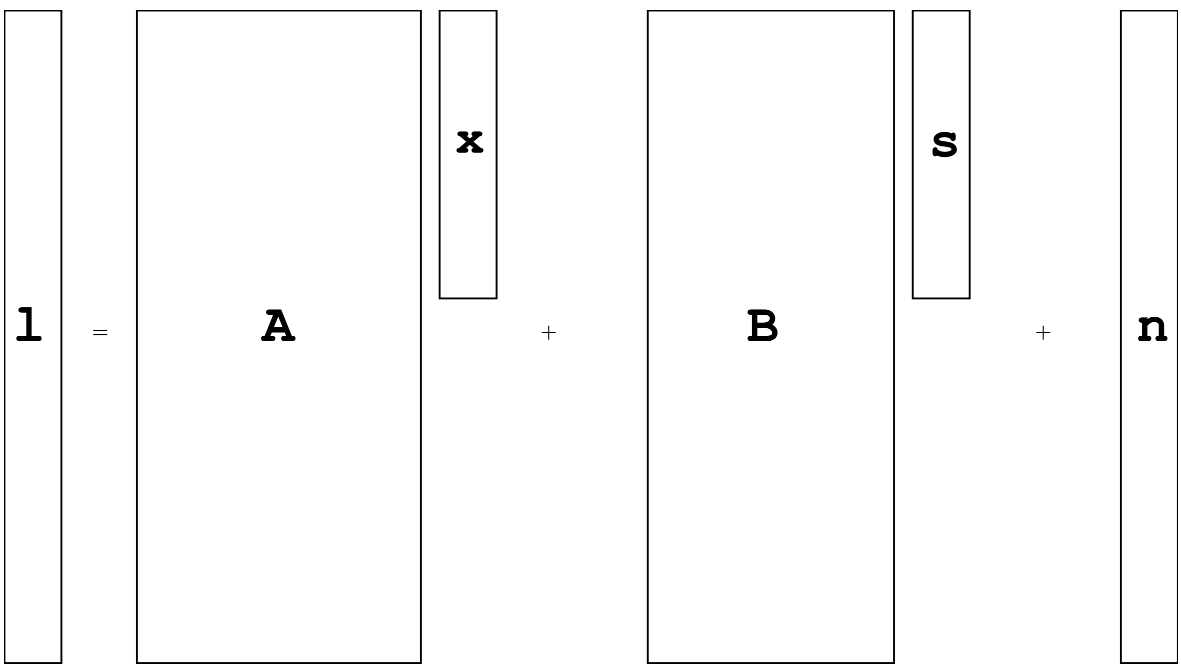
# The data
# A summery

- **Leveling**
  - *Relativ land uplift values between nodal points.*
  - *Some points are measured one time only.*
  - *Influenced by the rise of the geoid*
- **Tide gauges**
  - *Apparent land uplift values. Values relative to a rising mean see level.*
  - *Influenced by the rise of the geoid*
- **GPS-stations**
  - *'Absolute' values observed in a geodetic frame work we assume is stable.*
  - *Not influenced by the rise of the geoid*

# How to model the land uplift



# The model




| | | | |
|---|---|---|---|
| Where | l | = | **Observations** |
| | A | = | **Design matrix** |
| | X | = | **Unknown heights and trend coefficients** |
| | B | = | **Design matrix for the signals** |
| | s | = | **Signals (unknown land uplift.)** |
| | n | = | **Noise** |

# Observation equation

**For leveling**

$$l_i = -1\mathbf{h_a} + 1\mathbf{h_b} + \quad 1\mathbf{a} + x\mathbf{b} + y\mathbf{c} + xy\mathbf{d} + x^2\mathbf{e} + \ldots \qquad t_i\mathbf{s_a} + t_i\mathbf{s_b} \quad + n_i$$

**For tide gauges**

$$l_i = \qquad 1\mathbf{a} + x\mathbf{b} + y\mathbf{c} + xy\mathbf{d} + x^2\mathbf{e} + \ldots \qquad + 1\mathbf{s_i} \quad + n_i$$

**For GPS-stations**

$$l_i = \qquad 1\mathbf{a} + x\mathbf{b} + y\mathbf{c} + xy\mathbf{d} + x^2\mathbf{e} + \ldots + 1\mathbf{GPS_{const}} + u_{appr}\mathbf{GPS_{scale}} + 1\mathbf{s_i} + n_i$$

Where

| | | |
|---|---|---|
| $h_a$ | = | the height of the start point of the leveling line |
| $h_b$ | = | the height of the end point of the leveling line |
| a – e | = | coefficients on the trend surface. Only the five first are listed here. |
| $GPS_{const}$ | = | unknown constant in the GPS-rates |
| $GPS_{scale}$ | = | unknown scale factor in the GPS-rates |
| $s_a$ | = | signal in the start point of the leveling line |
| $s_b$ | = | signal in the end point of the leveling line |
| $s_i$ | = | signal in the station point |
| $x,\ y$ | = | coordinates of the points involved |
| $t_i$ | = | reference year – observation year |
| $u_{appr}$ | = | Approximated uplift ( = the observed GPS-rate) |
| $n_i$ | = | Noise |
| $l_i$ | = | An observation |

# The ordinary solution

$$\hat{\mathbf{x}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{C}_{\mathbf{xx}}{}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{C}_{\mathbf{xx}}{}^{-1}\mathbf{l}$$

$$\hat{\mathbf{s}} = \mathbf{C}_{\mathbf{ss}}\mathbf{B}^{\mathrm{T}}\mathbf{C}_{\mathbf{xx}}{}^{-1}\left(\mathbf{l} - \mathbf{A}\hat{\mathbf{x}}\right)$$

Where:

$$\mathbf{C}_{\mathbf{xx}} = \mathbf{C}_{\mathbf{ll}} + \mathbf{B}\mathbf{C}_{\mathbf{ss}}\mathbf{B}^{\mathrm{T}}$$

# *Schwarz method*

$$
\begin{bmatrix} N_{11} & N_{12} \\ N_{12} & N_{22}+C_{ss}^{-1} \end{bmatrix} \begin{bmatrix} X \\ S \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}
$$

**Where**

- **X - Unknown heights and trend surface**
- **S - Unknown land uplift**
- **$N_{11}$ - Normal equation matrix for X**
- **$N_{12}$ - Normal equation matrix for S**
- **$C_{ss}^{-1}$ - Inverse Co-variance matrix**

# Variance component estimation

- Four types of input to the normal equations
  - Leveling
  - Tide gauges
  - GPS-rates
  - Inverse co-variance matrix (artificial observations)

- The leveling observations are separated into eight groups
  - New and old Norwegian
  - 1. 2. and 3. Swedish leveling
  - 1. 2. and 3. Finnish leveling

- For each group we calculate:
  - Sum of squares $\mathbf{n}_i^T \mathbf{C}_i^{-1} \mathbf{n}_i$
  - Redundancy $\mathbf{r}_i$
  - Variance component $\hat{\sigma}_{0i}^2 = \frac{\mathbf{n}_i^T \mathbf{C}_i^{-1} \mathbf{n}_i}{\mathbf{r}_i}$

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# Results of the variance component estimation

- **Calculated standard deviations**
  - Leveling data:

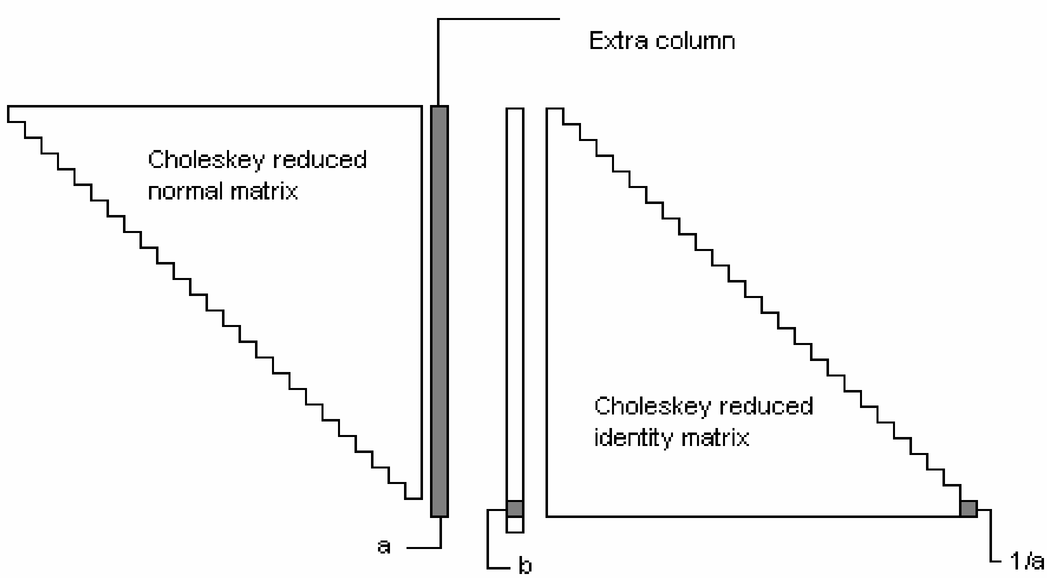
| | |
|---|---|
| Old Norway: | 1.3 mm/km |
| New Norway: | 1.1 mm/km |
| Finland 1.: | 1.1 mm/km |
| Finland 2.: | 0.9 mm/km |
| Finland 3.: | 0.8 mm/km |
| Swedish 1.: | 2.0 mm/km |
| Swedish 2.: | 1.4 mm/km |
| Swedish 3.: | 1.1 mm/km |

  - GPS-rates (average): 0.5 mm/yr
  - Tide gauges (average): 0.1mm/yr
  - Artificial observation: 0.3mm/yr (0.6)

# Test for outliers

**Multiple T-test:**
**Each observation is tested for outliers individually by setting an unknown parameter $\nabla$ into the normal equation system.**

**If the test value $\nabla / m_{\nabla}$ is higher than 3 we may claim an outlier.**




**After reduction of the extra column:**

$$\nabla = b/a$$

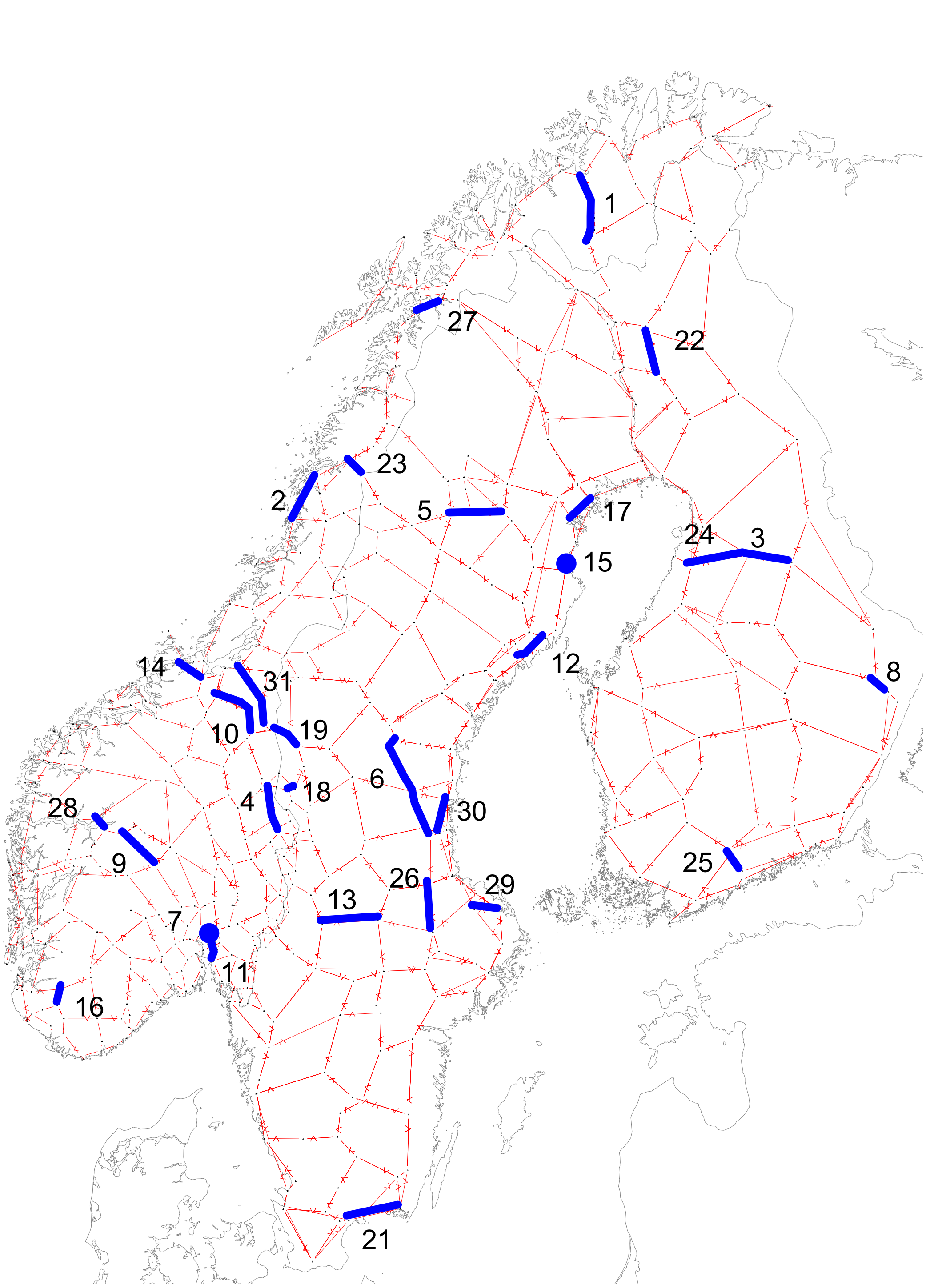
$$m_{\nabla} = m_0 \sqrt{q_{\nabla\nabla}} = \sqrt{((\sum pvv - b^2)/(ndf-1))}/a = m_0'/a$$

$$\nabla / m_{\nabla} = b/ m_0'$$

# Results
# of the test of outliers I

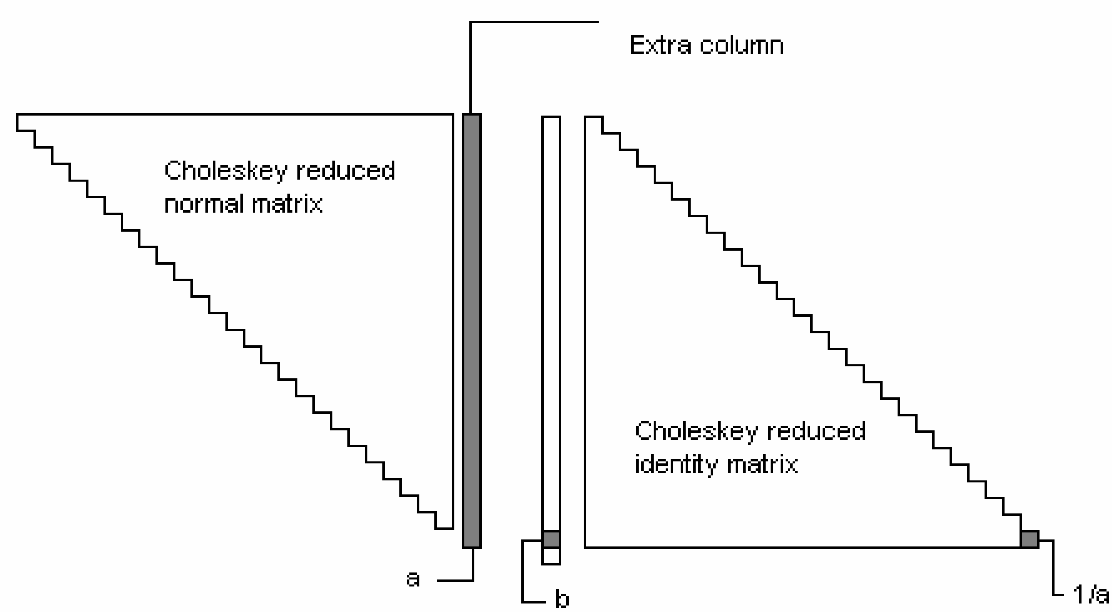
- ***Rejected outliers***
  - ***Norway***
    - ***Old:*** ***7***
    - ***New:*** ***6***
  - ***Sweden***
    - ***1. Leveling:*** ***6***
    - ***2. Leveling:*** ***2***
    - ***3. Leveling:*** ***3***
  - ***Finland***
    - ***1. Leveling:*** ***2***
    - ***2. Leveling:*** ***1***
    - ***3. Leveling:*** ***2***
  - ***Tide gauges***
    - ***Oslo*** ***0.9 mm/yr***
    - ***Furuøgrund*** ***0.9 mm/yr***
  - ***No GPS-rates are rejected***

# Results
# of the test of outliers II



# Internal reliability

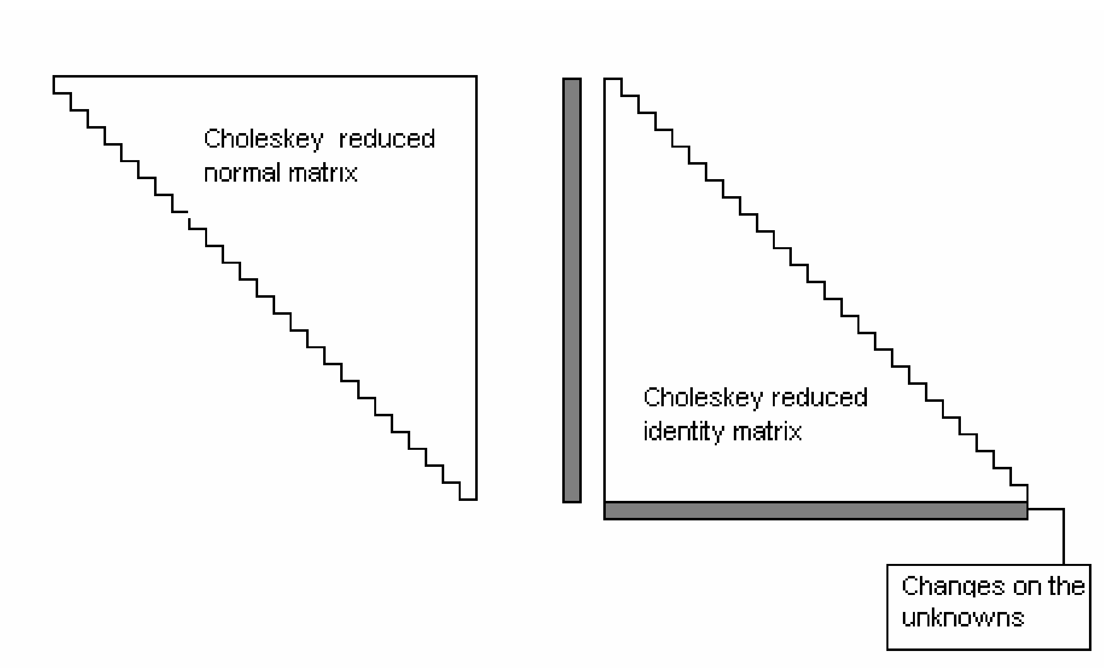
- ”A measure for how well the observations in a network mutually control each other” (Baarda: “The ability to detect gross errors”.)
  - Greatest remaining outlier [$\nabla$-2m$_\nabla$, $\nabla$+2m$_\nabla$]
    - Affected by network design, weights and observations.
  - Redundancy: a number between 0 an 1
    - Affected by network design and weights only.
    - In simple cases, such as levling, it turns out that the redundancy $r_i = a^2/p_i$ where $p_i$ is the weight



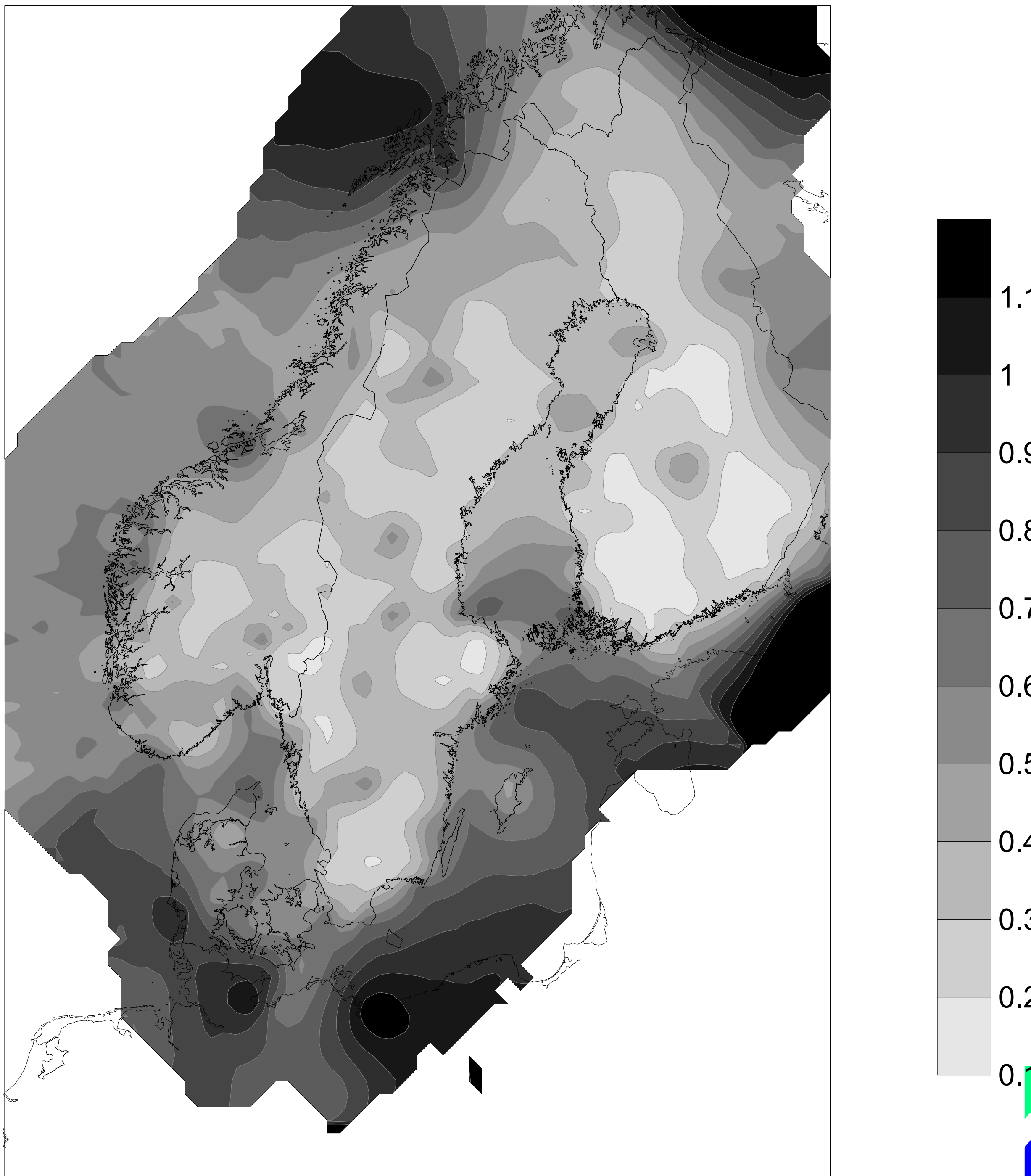
# External reliability

- "Maximum influence of a remaining outlier on the unknowns, or on a function of them."
- In our case we are interesting in the land uplift which means influences on the trend coefficients and on the signals

**New constant columns have to be Choleskey reduced:**



# Reliability of land uplift model

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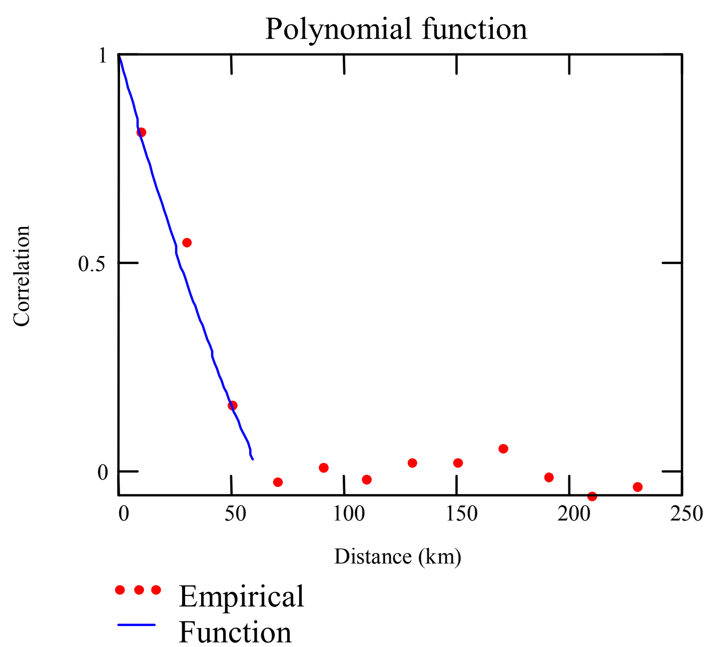
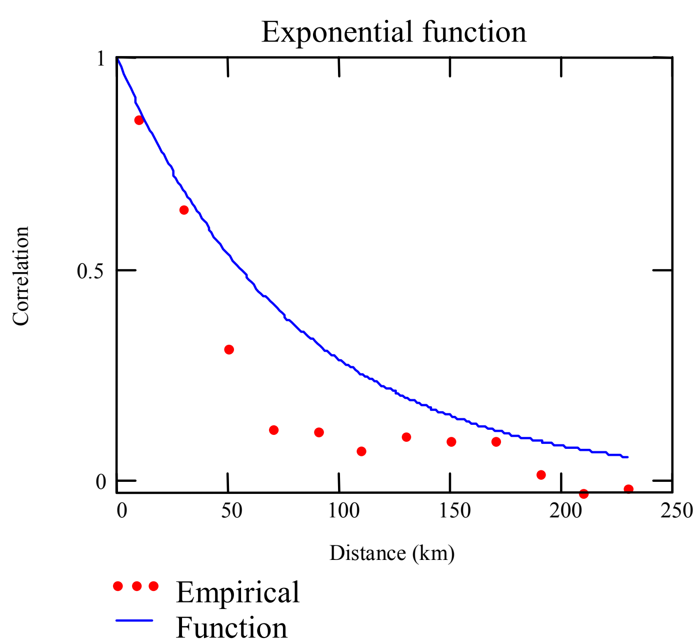
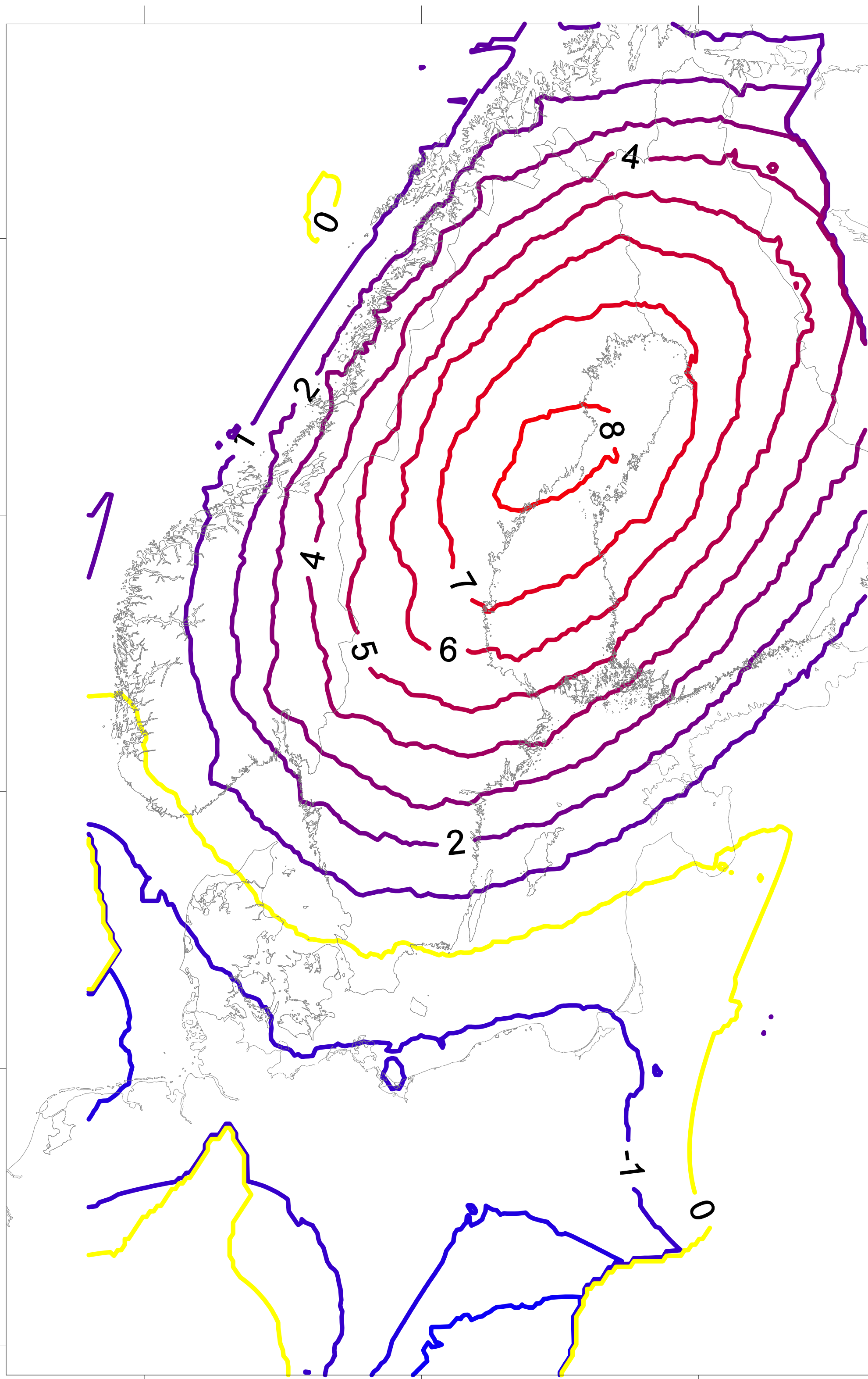
# The co-variance model

$$F(X) = 10D^2 - 8D + 1$$



# Land uplift

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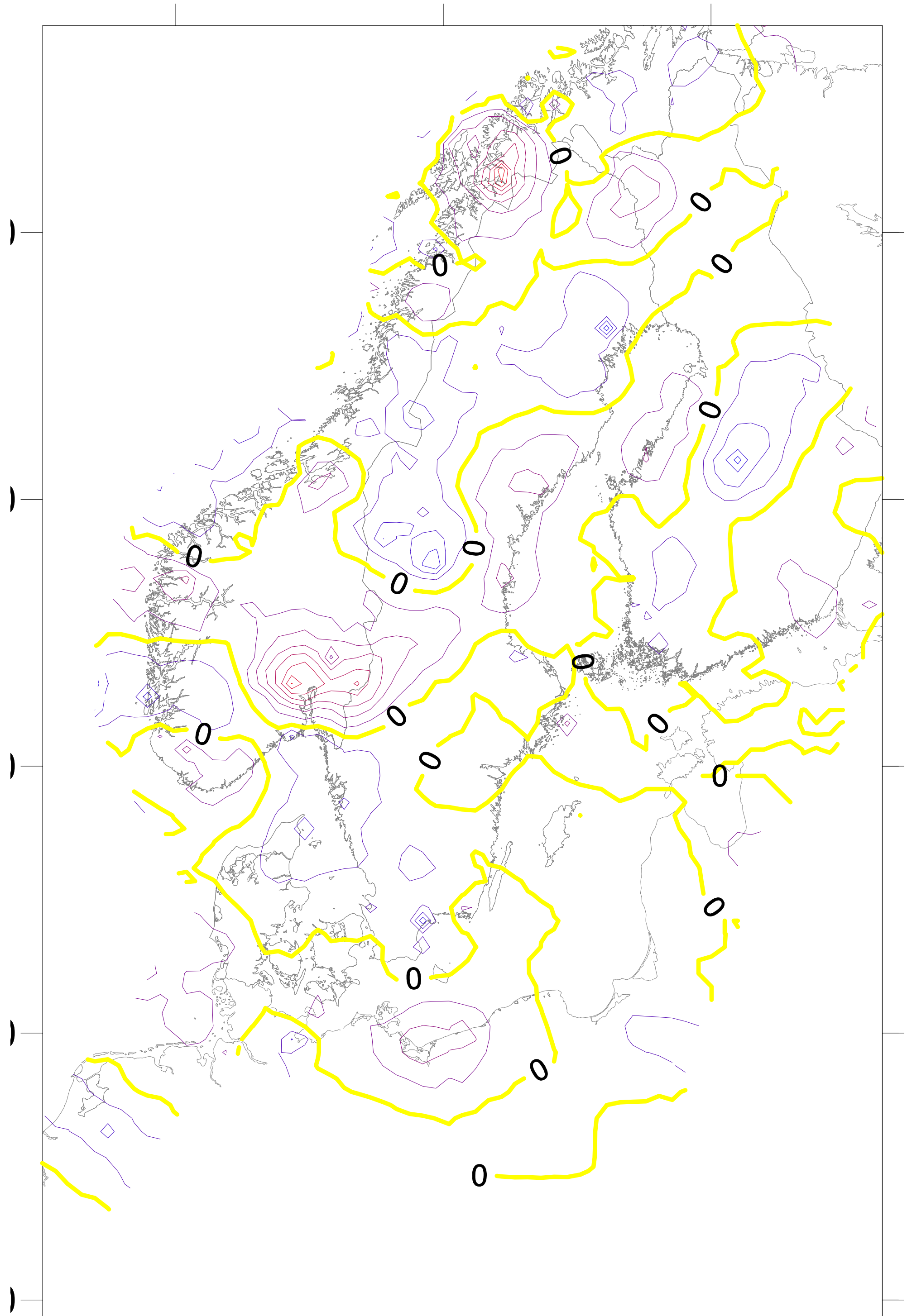
# Land uplift signals

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# The fit of the GPS-rates

- Constant and scale
  - -1,32 mm/yr +/- 0.14 mm/yr
  - 5.7% of the absolute land uplift +/- 2.3%

- Residuals (mm/yr)
  - Five larges

| | |
|---|---|
| WROC (Poland) | 1,54 |
| BUDP (Denmark) | -1,24 |
| OSLS (Norway) | 1,22 |
| LAMA (Poland) | -1.05 |
| TRDS (Norway) | -0.99 |

  - In total

| | |
|---|---|
| RMS | 0,44 |
| Mean | -0,01 |

- Outlier test
  - No station rejected
  - Mar6 (Sweden) suspect

| | |
|---|---|
| T-value | 2,7 |
| Outlier value | -0,78 mm/yr |

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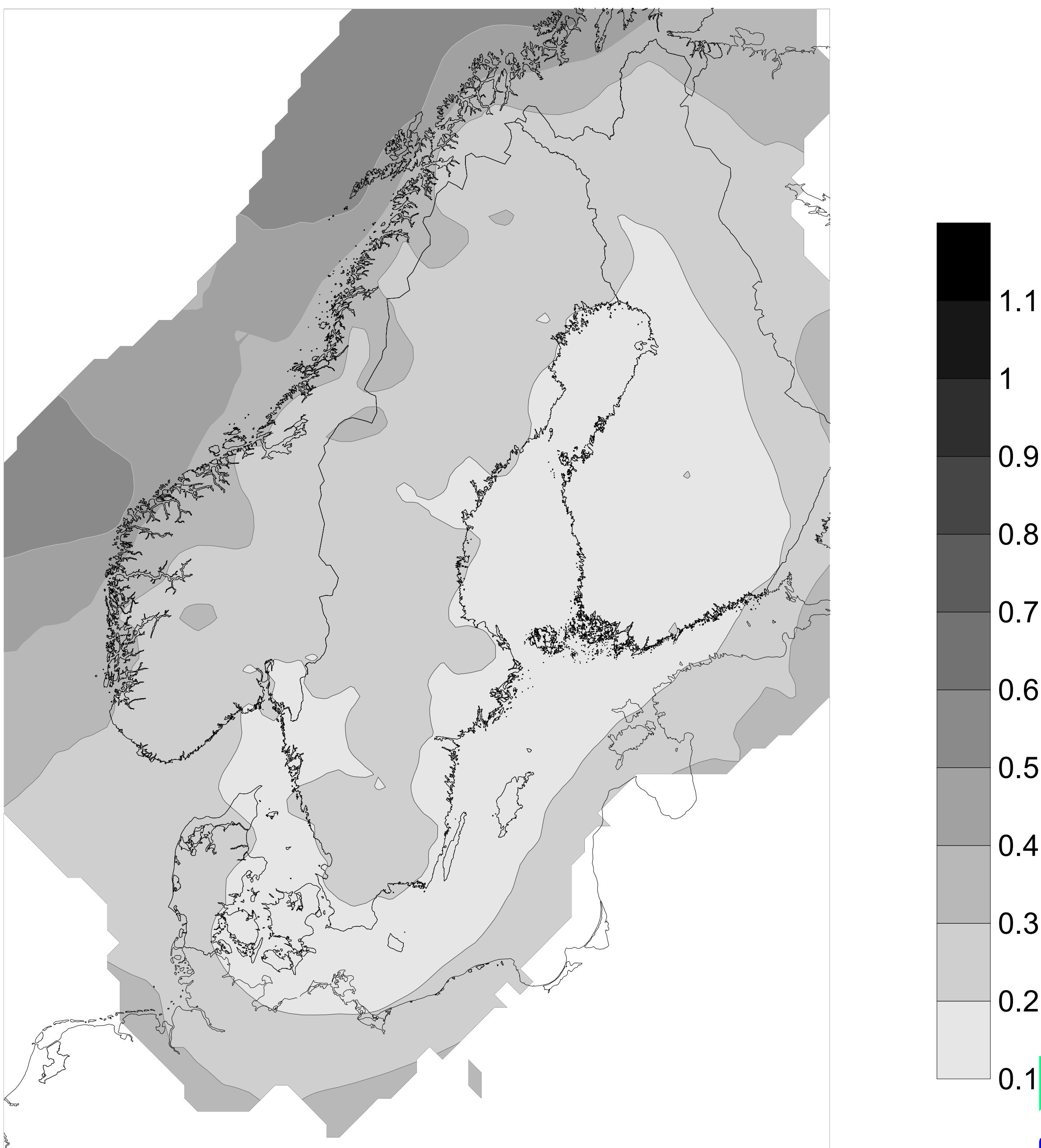
# Standard deviations on the uplift values

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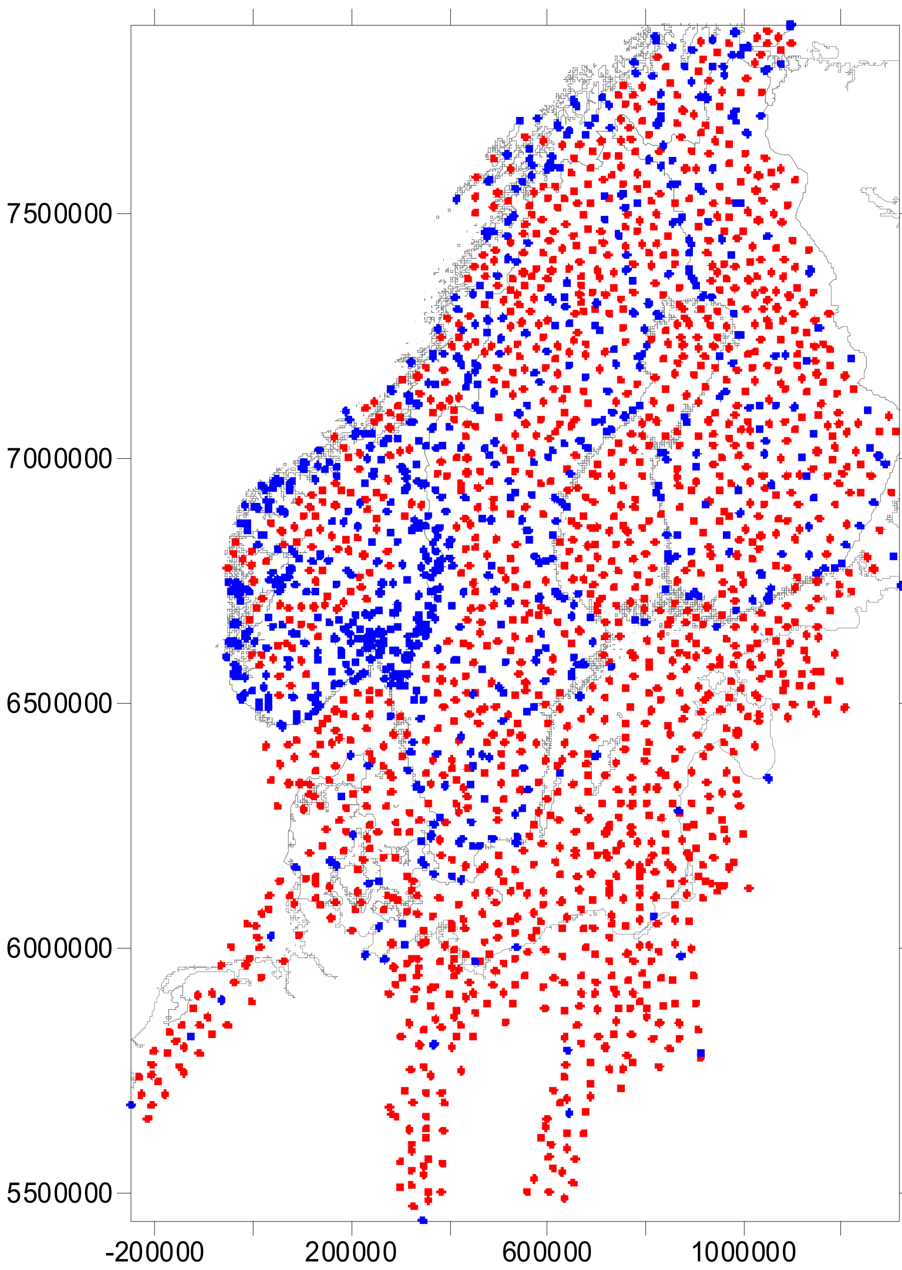
# Gridding of the model

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