GEOPOTENTIAL FOR SPECIFYING RELATIVISTIC ATOMIC TIME SCALE AND GLOBAL VERTICAL REFERENCE SYSTEM

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The definition of Terrestrial Time (TT), which is used by all space technique, requires that the geopotential W_0 value is specified at the mean sea level (MSL):

- Terrestrial Geocentric Coordinate Time (TCG) differs from by a
constant rate (L_G)
- ITRF2000 scale is now defined in TCG

$$dTT / dTCG = 1 - W_0 / c^2 = 1 - L_G$$

(c is the speed of light)

L_G is required for transformation of TT, based on SI seconds on the surface $W=W_0$, to TCG. I.e.,

$$TCG - TT = L_G \times (MJD - 43144) \times 86400s$$

Where MJD is modified Julian Date of International Atomic Time (TAI),

$$TT = TAI + 32.184s$$

The above transformations, are required in geodesy, since all the modern space technique use TT, not TCG, I.e., the implied ITRF scale ratio is

$$ITRF(TCG) / ITRF(TT) = 1 + L_G$$

IAU (International Astronomical Union) adopted L_G as a defining constant (IAU XXIV(2003), Resolution B19):

$$L_G = 6.969\,290\,13 \cdot 10^{-10},$$

however, it is based on the proposed value (see below) of

$$W_0 = 62\,636\,856.0 m^2 s.$$

<u>Thus, W_0 is needed for:</u>

- **time keeping**
- **ITRF2000(TCG) scale**
- **a definition of a Global Vertical Reference System**

(GVRS), or a global vertical datum (see below).

1st STEP: ADOPTING A REFERENCE VALUE W_0

- THEORETICALLY, W_0 CAN BE ARBITRARY
- PRACTICALLY, IT IS ADVANTAGEOUS WHEN IT CORRESPONDS TO THE MEAN OCEANS

$$\int_{S_0} \left(W - W_0\right)^2 \mathrm{d}S_0 = \text{minimum}$$

(S_0 stands for the World Ocean Surface)

Table 1. Yearly mean values of W_0 and $R_0 = GM/W_0$; based on and EGM96 and Topex/Poseidon altimetry, no IB corrections applied.

Year	Number of points	W_0 [$m^2 \cdot s^{-2}$]	rms [$m^2 \cdot s^{-2}$]	R_0 [m]	rms [m]
1993	203 856	62 636 856.157	0.005	6 363 672.5452	0.000 5
1994	206 973	62 636 856.168	0.005	6 363 672.5440	0.000 5
1995	205 746	62 636 856.163	0.005	6 363 672.5445	0.000 5
1996	203 960	62 636 856.158	0.005	6 363 672.5450	0.000 5
1997	216 757	62 636 856.157	0.005	6 363 672.5451	0.000 5
1998	206 803	62 636 856.162	0.005	6 363 672.5446	0.000 5
1999	203 764	62 636 856.162	0.005	6 363 672.5446	0.000 5
2000	208 814	62 636 856.157	0.005	6 363 672.5452	0.000 5
2001	208 402	62 636 856.151	0.005	6 363 672.5457	0.000 5
2002	197 951	62 636 856.149	0.005	6 363 672.5460	0.000 5
1993 -2002	2 063 026	62 636 856.158	0.002	6 363 672.5450	0.000 2

($R_0 = GM/W_0$ - the geopotential scale factor)

ADVANTAGEOUS PROPERTIES OF W_0:

- **SUFFICIENTLY STABLE (see Table 1)**
- **INVARIANT WITH RESPECT TO THE TIDAL REFERENCE SYSTEM(see below)**

ACTUAL ACCURACY:

±(0.3-0.5) m^2s^{-2} ±(3-5) cm

DUE TO THE LIMITING FACTORS:

CALIBRATION ERROR OF TOPEX/POSEIDON ALTIMETER SYSTEM ~ 3-5 cm

CONSEQUENTLY, THE ROUNDED VALUE RECOMMENDED (Burša et al., see SSG GGSA papers, especially 2002)

$$\underline{W_0=(62\ 636\ 856.0\pm 0.5)\ m^2s^{-2}} \quad (1)$$

FOR A GVRS, BESIDES W_0 (1), THREE OTHER PRIMARY FUNDAMENTAL GEODETIC PARAMETERS SHOULD BE ADOPTED:

$$GM = (398\ 600\ 441.8 \pm 0.8) \times 10^{6}\ \mathrm{m^3 \cdot s^{-2}} \qquad (2)$$

$$\omega = 7\ 292\ 115 \times 10^{-11}\ \mathrm{rad \cdot s^{-1}} \qquad (3)$$

$$J_2 = (1\ 082\ 635.9 \pm 0.1) \times 10^{-9};$$

…in the zero-frequency tide system, (4.1)

or

$$J_2 = (1\ 082\ 666.7 \pm 0.1) \times 10^{-9};$$

…in the mean tide system, (4.2)

or

$$J_2 = (1\ 082\ 626.7 \pm 0.1) \times 10^{-9}.$$

…in the tide-free system. (4.3)

SPECIFYING THE TIDE REFERENCE SYSTEM (the zero, mean, or tide-free) IS REQUIRED FOR A DEFINITION OF THE GVRS !

AFTER ADOPTING THE FOUR FUNDAMENTAL CONSTANTS (1) - (4) THE NORMAL GRAVITY POTENTIAL AND THE EQUIPOTENTIAL ELLIPSOID E_0 IS UNIQUELY DETERMINED:

$$E_0 = E_0(GM, \omega, W_0, J_2) \qquad (5)$$

ON THE BASIS OF THE PIZZETTI's THEORY

THEN, THE THREE UNIQUELY DETERMINED, DERIVED PARAMETERS ARE :

$$a = a\,(GM, \omega, W_0, J_2) \qquad (6)$$

$$\alpha = \alpha\,(GM, \omega, W_0, J_2) \qquad (7)$$

$$\gamma_e = \gamma_e\,(GM, \omega, W_0, J_2) \qquad (8)$$

Table 2. The derived parameters and W_0

TIDAL SYSTEM	a [m]	$1/\alpha$	W_0 [m^2s^{-2}]	γ_e [mGal]
zero	6378136.58	298.25645	62 636 856.0	978 032.672
mean	6378136.68	298.25234	62 636 856.0	978 032.687
tide-free	6378136.55	298.25769	62 636 856.0	978 032.667

AFTER ADOPTING THE REFERENCE W_0-value, THE GEOPOTENTIAL DIFFERENCES

$$\delta W_{0i} = W_0 - W_{0i}$$

for the ith LVD (local vertical datum) SHOULD BE DETERMINED

-A METHODOLOGY WAS DEVELOPED BY THE SSG GGSA AND WAS PRACTICALLY APPLIED

-ACCURACY DEPENDS ON THE RESOLUTION OF THE GEOPOTENTIAL MODEL (see EGM96R in Table 3 and Fig. 1)

-THERE MAY ALSO BE ADDITIONAL DISTORTIONS DUE TO SYSTEMATIC LEVELLING ERRORS etc.

Table 3. EGM96R, according to (NRC, 1997)

s [km]	EGM96R [mm]
10	400.0
100	260.0
200	150.0
400	90.0
600	55.0
1 000	30.0
2 000	10.0
4 000	2.5
6 000	1.0

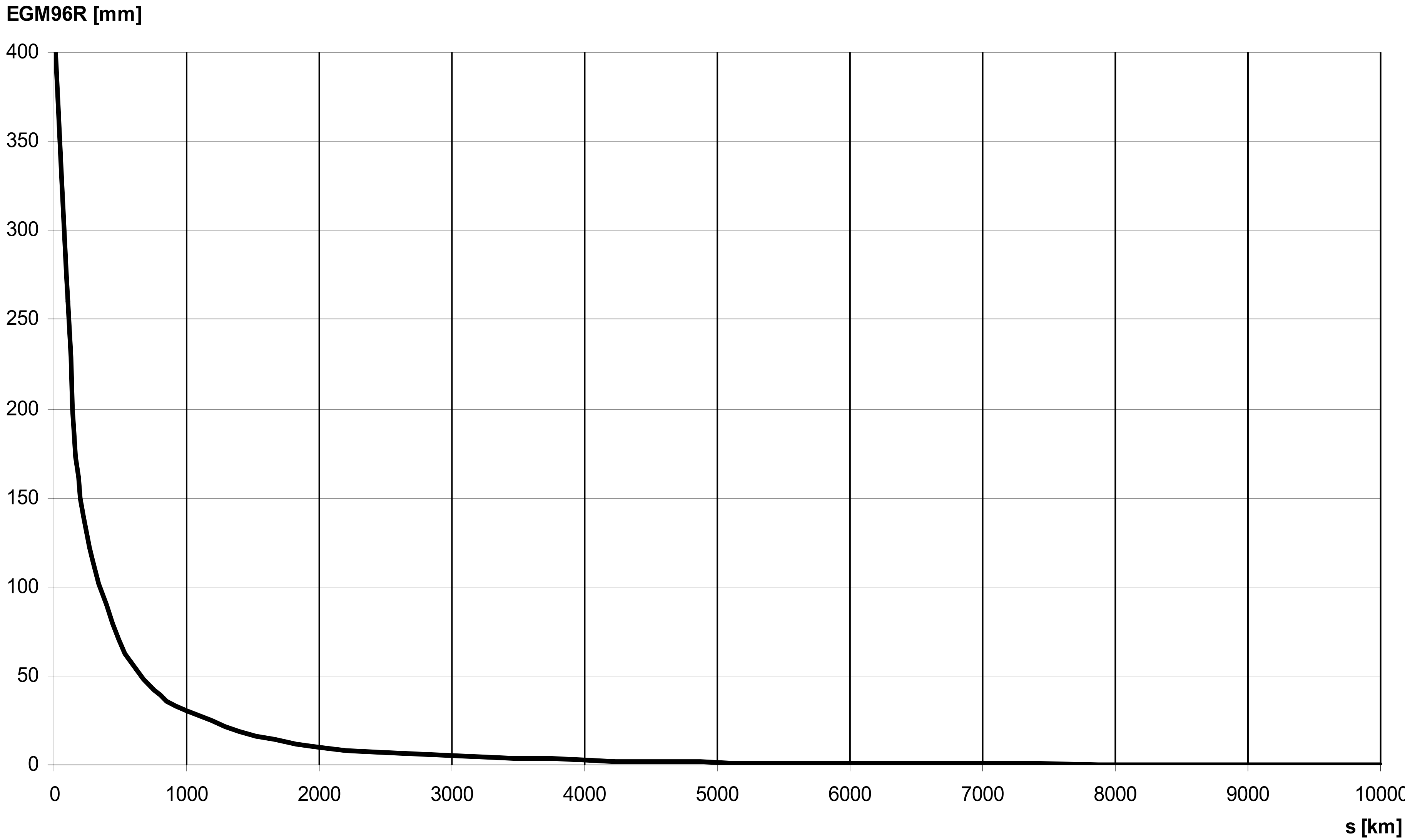

Figure 1. EGM96R, according to (NRC, 1997)

EXAMPLE OF A SUCCESSFUL APPLICATION OF The SSG GGSA METHODOLOGY:

- NAVD 88 (North American Datum 1988)
- SUFFICIENTLY LARGE AREA, COVERING THE TERRITORY OF U.S.A. AND CANADA
- NORMAL, TIDALLY CORRECTED, HEIGHTS AVAILABLE
- ACCURATE TIDE-FREE GPS COORDINATES AVAILABLE (ITRF is tide-free!)
- TIDE-FREE EGM96

Table 4. Geopotential values W_{0i} at the local vertical datums (LVDs); δH_{0i} is the vertical shift of the LVD origin, related to the reference surface $W=W_0$. EGM96R is the estimated resolution error of EGM96, according to NRC 1997

Territory	LVD i	Number of GPSLS	EGM96R [cm]	W_{0i} $[m^2s^{-2}]$	$W_{0i} - W_0$ $[m^2s^{-2}]$	δH_{0i} [m]
USA	NAVD88	5168	1.0	62 636 861.27 ± 0.51	+5.27 ± 0.11	-0.54 ± 0.01
Canada	NAVD88	1311	1.4	62 636 861.54 ± 0.53	+5.54 ± 0.17	-0.56 ± 0.02

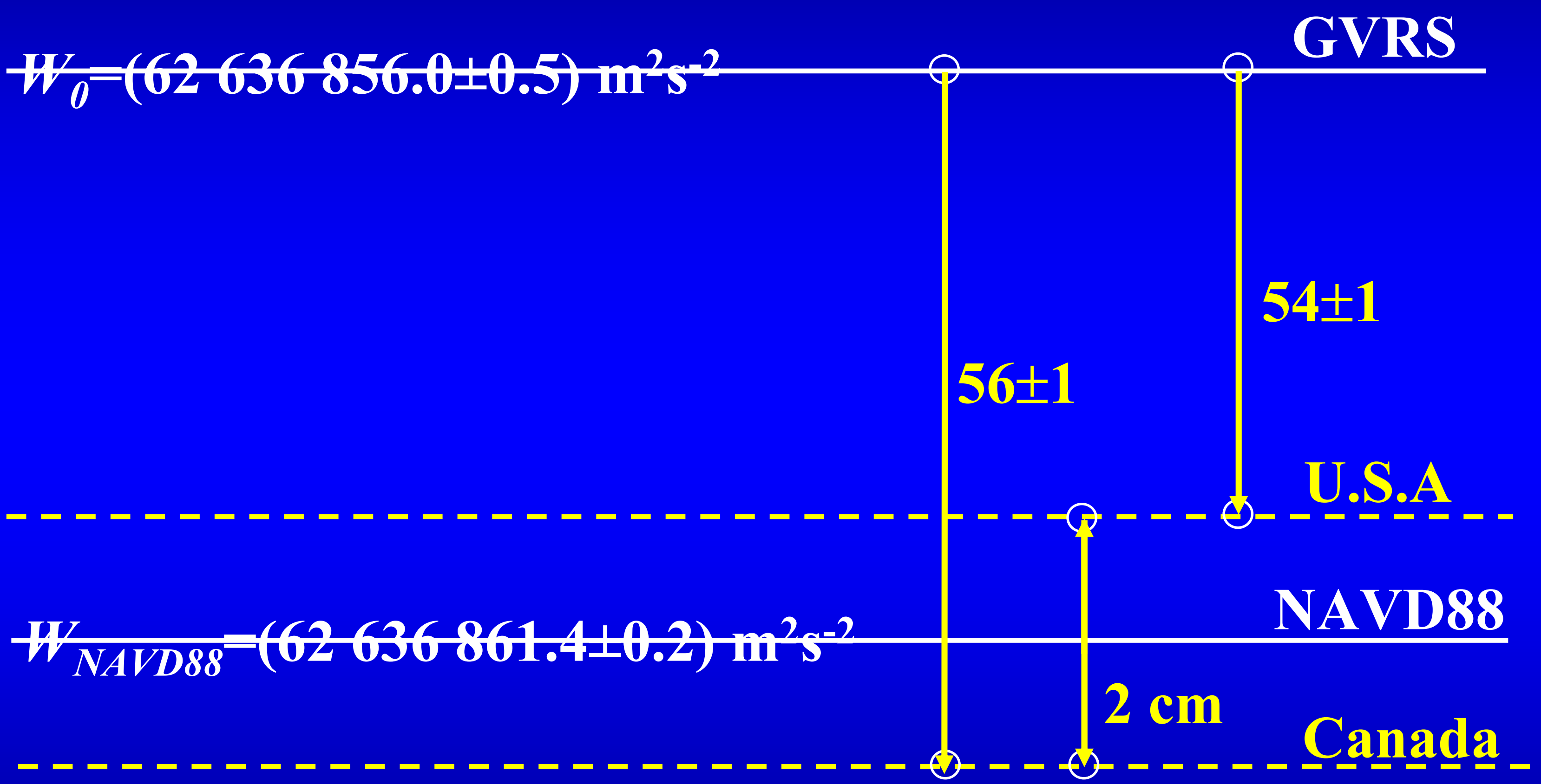

Figure 2. Vertical shifts (in cm) of LVD realizing the NAVD88 vertical system

DISCUSIONS, CONCLUSIONS

GVRS CAN BE UNIQUELY SPECIFIED BY A REFERENCE W_0-VALUE

-W_0 representing the mean ocean surface, is stable (Tab. 1) and nowadays easily accessible through satellite altimetry

- W_0 is practically the same for all recent gravity field models (EGM96, CHAMP and GRACE models, etc.)

- $W_0 = (62\ 636\ 856.0 \pm 0.5)\ m^2s^{-2}$ *was already used for the fundamental constant L_G, defining relativistic time and ITRF scale, this is why it is also proposed for the GVRS*

- ***tidal reference system of the* GVRS *must be specified*: AN ABSOLUTE NECESSITY!**

- ***when connecting LVD's, the area covered by LVD's should be sufficiently large, the EGM resolution should be sufficiently high (see Table 3 and Fig.1)***

-*in Europe: the most suitable example is $LVD_{KRONSTADT}$, since it covers the largest area of Europe*

-*on the basis of EUVN height solution* **(Ihde and Augath: The Vertical System for Europe, 2001)** *we determined* **:**

$$(W_0)_{NAP}=(62\ 636\ 857.55 \pm 0.61)\,m^2s^{-2}$$

-*however, the actual accuracy is lower than the above estimate, because EUVN data is not uniform regarding the tide reference system*

ALL THE DATA USED SHOULD BE UNIFIED IN REGARDS TO THE TIDE REFERENCE SYSTEM!

NOTE THAT CURRENTLY:

THE GPS COORDINATES:	**TIDE-FREE**
EGM96:	**TIDE-FREE**
HEIGHTS:	**NOT UNIQUE**
GRAVITY:	**MEAN- TIDE**

SSG GGSA papers on GVRS, GVRF, and W_0

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