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THE DANISH NATIONAL SPACECENTER IS A RESEARCH CENTER UNDER THE MINISTRY OF SCIENCE, TECHNOLOGY AND INNOVATION

Geoid determination and fit to GPS/Levelling

by

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Introduction

Department of Geodynamics

Until 1/1/2005: Kort og Matrikelstyrelsen (National Survey and Cadastre)

From 1/1/2005: Danish National Space Center

Some work duties:

Gravity network (Denmark, Greenland, Faeroe Islands)

Geoid modelling (NKG, UK+Ireland, Malaysia, Taiwan, Mongolia, Poland, GOCINA, …)

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Danish National Space Center

New research organization formed 1/1 2005

- 70 employees
- Merger of Danish Space Research Institute and KMS geodesy (20 persons)
- KMS seismology -> GEUS

Due to government rearrangement of sector research

- Astrophysics
- Sun-earth phys.
- Planetery phys.
- Geodesy
- Geodonamics
- Technology

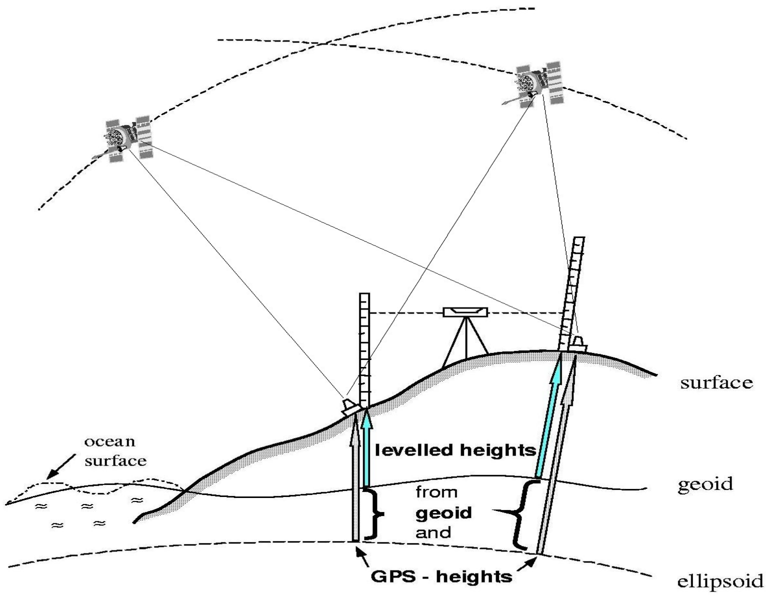
Major projects:
SWARM – ESA magnetfield mission
NUStar – NASA x-ray observatory
PLANCK – fundamental physics mission
Geodesy/Earth Observation:
EU/ESA projects: GreenIce, SITHOS, GOCINA, CRYOSAT, ArcGICE, Mongolia ..

Why the geoid?

Land: Height determination with GPS:

$$H_{orthometric} = h_{ellipsoidal} - N$$

N geoid height. Challenge: *Getting geoid to 1 cm.*



Geoid determination basics

Geoid: Level surface of global undisturbed oceans ..

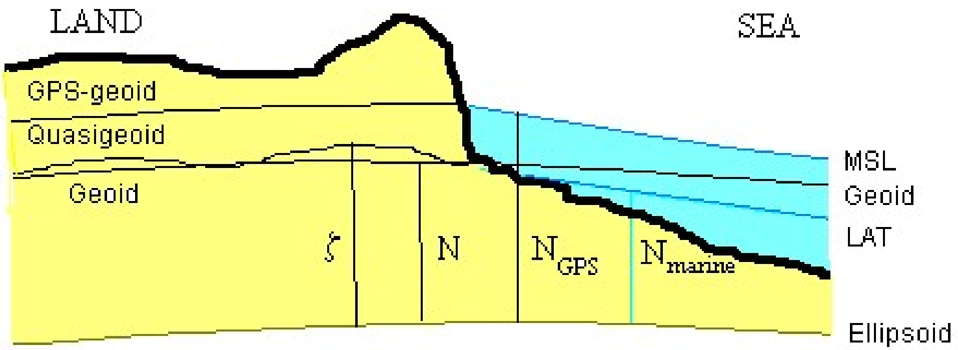
Complications:

”Geoid” is *global* equipotential surface in geopotential ($W = W_0$),

but heights refer to *local* reference:

- Mean Sea Level (MSL) – land
- Lowest Astronomic Tide (LAT) – sea

The ”cm-geoid” always refers to a specific datum .. and is not necessarily a ”true” geoid



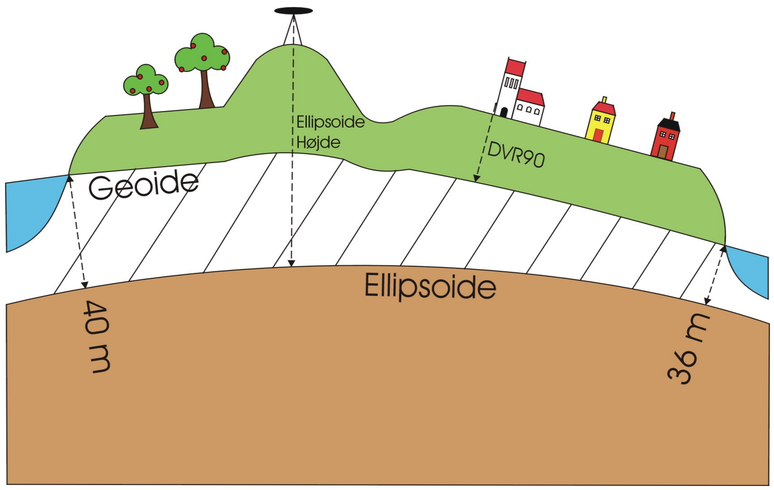
Geoid determination basics (2)

Geoid: Reference surface for GPS: $H_{sea\text{-}level} = h^{GPS} + N$

H: local datum (tide gauge link)

h: WGS84 (ITRF)

N: gravimetric geoid (from gravity and satellites)



In local datum:

$H = h + N + \varepsilon$

ε takes into account *datum, movements* and *errors*

At GPS-levelling point: $\varepsilon = h - H + N$

$N' = N + \varepsilon$ is the ***GPS geoid*** *... gives heights in national levelling system when using GPS*

Gravimetric geoid – theory

Anomalous gravity potential *T* is split into 3 parts:

$$T = T_{EGM} + T_{RTM} + T_{res}$$

T_{EGM} – EGM (Earth's Gravity Model) f.ex. EGM96 global spherical harmonic model complete to degree and order 360. Now days: GRACE 2s model.

T_{RTM} – residual terrain effect (RTM)

T_{res} – residual (i.e. unmodelled) regional/local gravity effect

Gravimetric geoid – theory (cont.)

Final goal is the geoid *N,* i.e. "inside masses" (*H=0*):

$$N = \frac{T(\varphi, \lambda, 0)}{\gamma(\varphi, \lambda, 0)}$$

(T non-harmonic inside masses)

Modelled directly: the *quasi-geoid* ζ, i.e. "the geoid on topography":

$$\zeta = \frac{T(\varphi, \lambda, H)}{\gamma(\varphi, \lambda, H)}$$

where γ is the normal gravity, ϕ is the latitude, λ is the longitude and H is the (orthometric) height.

Gravimetric geoid – theory (cont.)

The *quasi-geoid* ζ is split into 3 parts:

$$\zeta = \zeta_{EGM} + \zeta_{RTM} + \zeta_{res}$$

ζ_{EGM} – For example. EGM96 global spherical harmonic model complete to degree and order 360

ζ_{RTM} – residual terrain effect (RTM)

ζ_{res} – residual (i.e. unmodelled) regional/local gravity effect.

Gravimetric geoid – theory (cont.)

Approximately, the relation between *the geoid N* and t*he quasi-geoid* ζ *is*:

$$\zeta - N = H_P - H_P^* \approx -\frac{\Delta g_B}{\gamma_0} H$$

ζ *-N* is, thus, the difference between the *orthometric* height H_P and the *normal* height H^*_P, Δg_B is the Bouguer anomaly and γ_0 is the normal gravity at the ellipsoid.

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Gravimetric geoid – theory (cont.)

ζ_{EGM}

For example, for the *global field (T_{EGM96})* the EGM96 spherical harmonic model (Lemoine et al., 1996), complete to degree and order N=360, can be used:

$$\zeta_{EGM} = \frac{GM}{R\gamma} \sum_{n=0}^{N} \left(\frac{R}{r}\right)^{n} \sum_{m=0}^{n} \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin\phi)$$

Now days, we use a merged model GRACE2s + EGM96.

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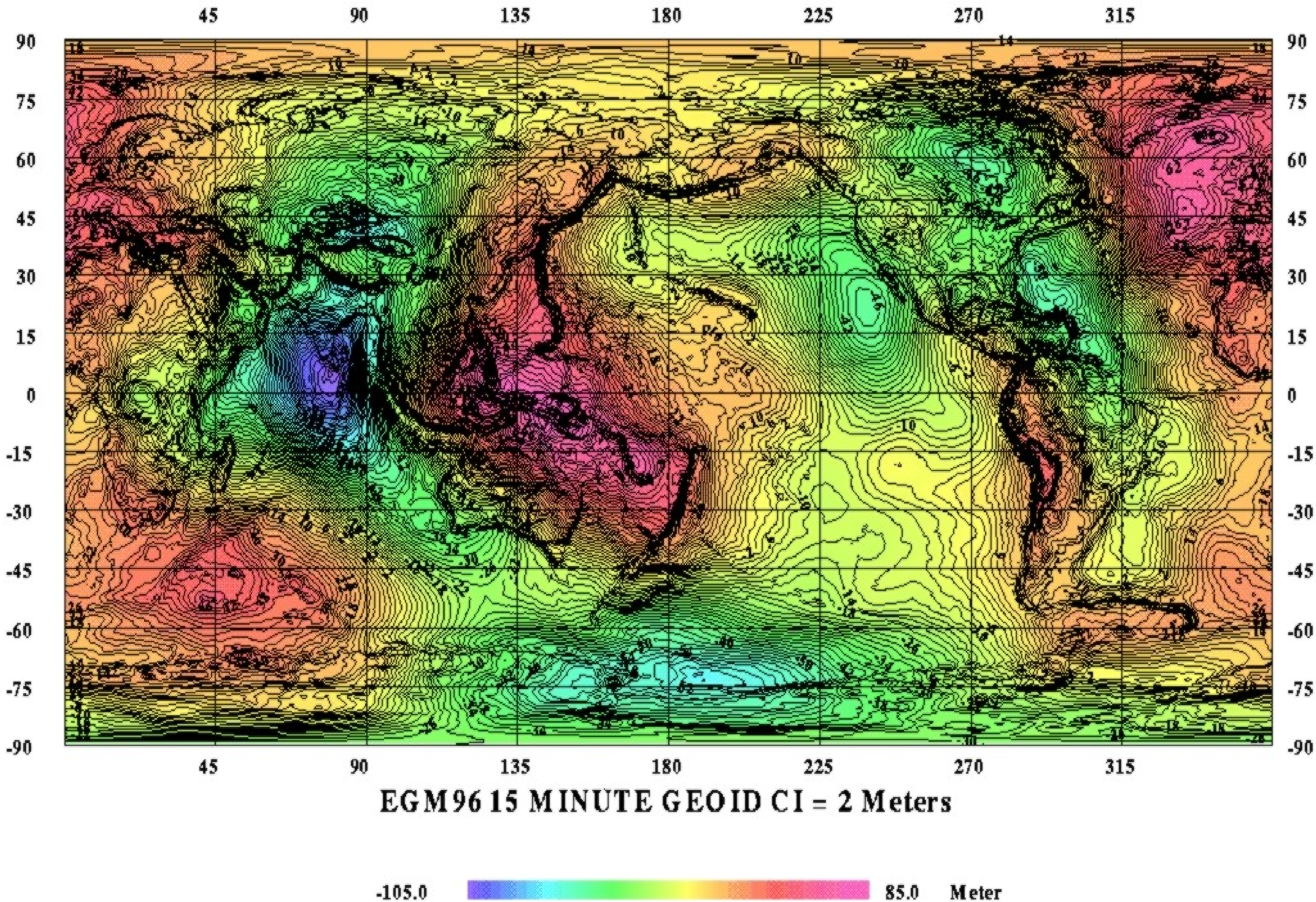
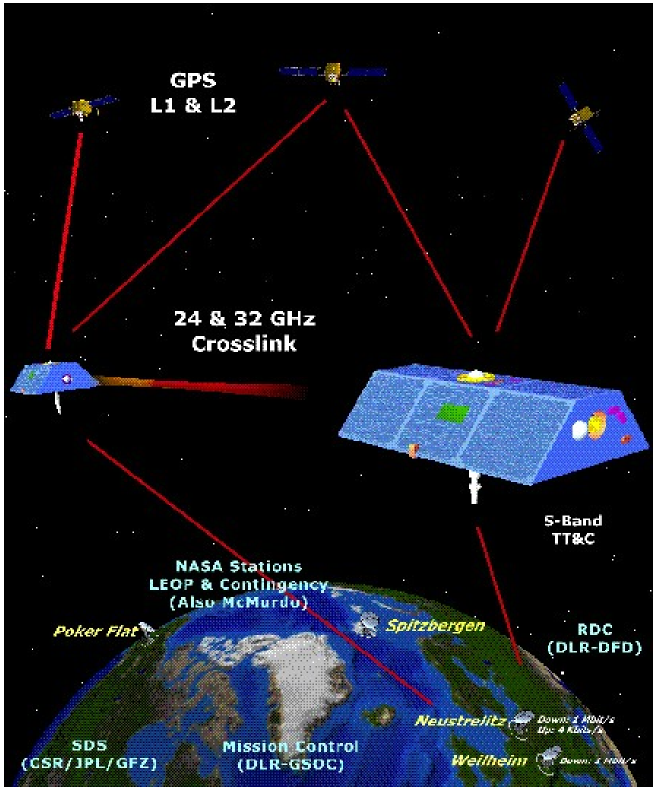
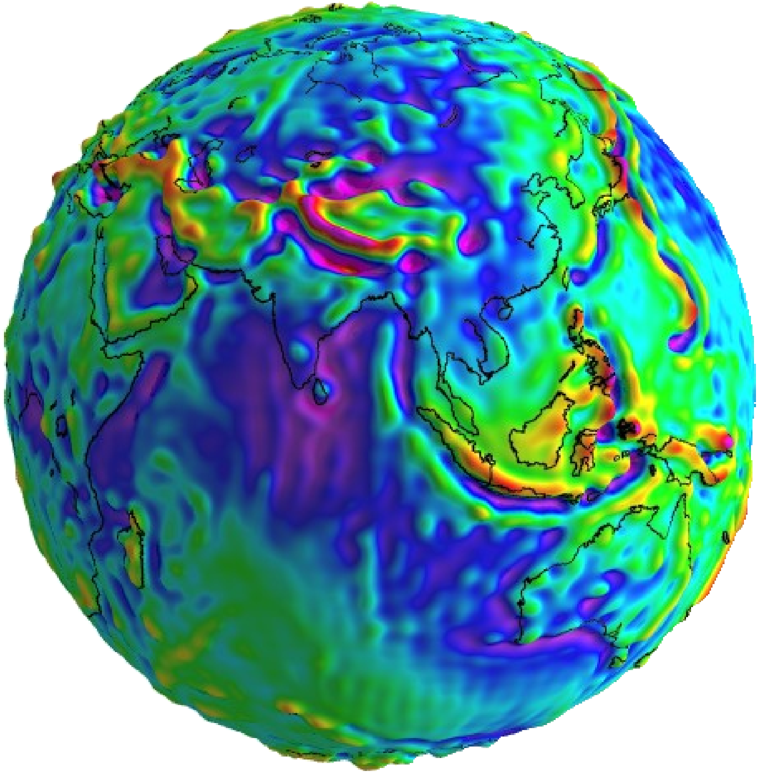


Satellite gravity: GRACE – launched 2002



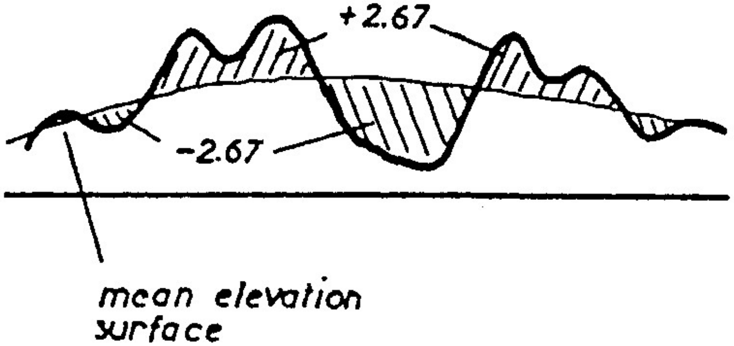


GRACE = Gravity and Climate Explorer; US/German satellite

Gravimetric geoid – theory (cont.)

ζ_{RTM}

Fourier-domain; 3rd order height term included



$$\varsigma_{RTM} = G\rho \int_{-\infty}^{\infty}\int \int_{z=h_{ref}(x,y)}^{z=h(x,y)} \frac{z - h_P}{[(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - h_P)^2]^{1/2}} dx_Q dy_Q dz_Q$$

$$\varsigma_{RTM} \approx G\rho[(h - h_{ref}) * \frac{1}{s} - 3h_P^2(h - h_{ref}) * \frac{1}{6s^3} - (h^3 - h_{ref}^3) * \frac{1}{6s^3} + 3h_P(h^2 - h_{ref}^2) * \frac{1}{6s^3}]$$

Iceland: glaciers treated correctly (Omang et al., 2001)

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Gravimetric geoid – theory (cont.)

ζ_{res}

Stokes' kernel modification (i.e. modified Wong-Gore method without "sharp edges"):

$$S_{\text{mod}}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n+1}{n-1} P_n \cos\psi$$

where

$$\alpha(n) = \begin{cases} 1 & \mathit{for} \quad 2 \le n \le N_1 \\ \dfrac{N_2 - n}{N_2 - N_1} & \mathit{for} \quad N_1 \le n \le N_2, \qquad n = 2,\ldots,N \\ 0 & \mathit{for} \quad N \ge n > N_2 \end{cases}$$

Gravimetric geoid – theory (cont.)

Stokes' kernel modification

2D- multiband spherical FFT method as implemented in SPFOUR (GRAVSOFT).

$$\zeta_{res} = \frac{1}{\gamma} F^{-1}[\, F\left(S_{\mathrm{mod}}\right) F(\Delta g_{res})]$$

where

$$\Delta g_{res} = \Delta g - \left(\Delta g_{EGM96} + \Delta g_{RTM}\right)$$

and *F* is the Fourier transform.

Criterion: The best fit to the misfit ε between the GPS-levelling data and the gravimetric geoid.

Fitting to GPS/Levelling

Fitting of a gravimetric geoid to a set of GPS geoid heights entails modelling the difference signal

$$\varepsilon = N_{lev} - N_{grav}$$

and adding the modelled ε-correction to the gravimetric geoid. In this way a new "geoid grid" is obtained which is "tuned" to the levelling and GPS datum in question.

$$\varepsilon = \varepsilon_{trend} + \varepsilon_{res}$$

where ε_{trend} is the "long wavelength" trend correction surface and ε_{res} is the "short and medium wavelength" residual.

Iceland geoid

NKG96

EGM: EGM96

spacing: $\Delta\varphi \times \Delta\lambda = 0.025^\circ \times 0.05^\circ$

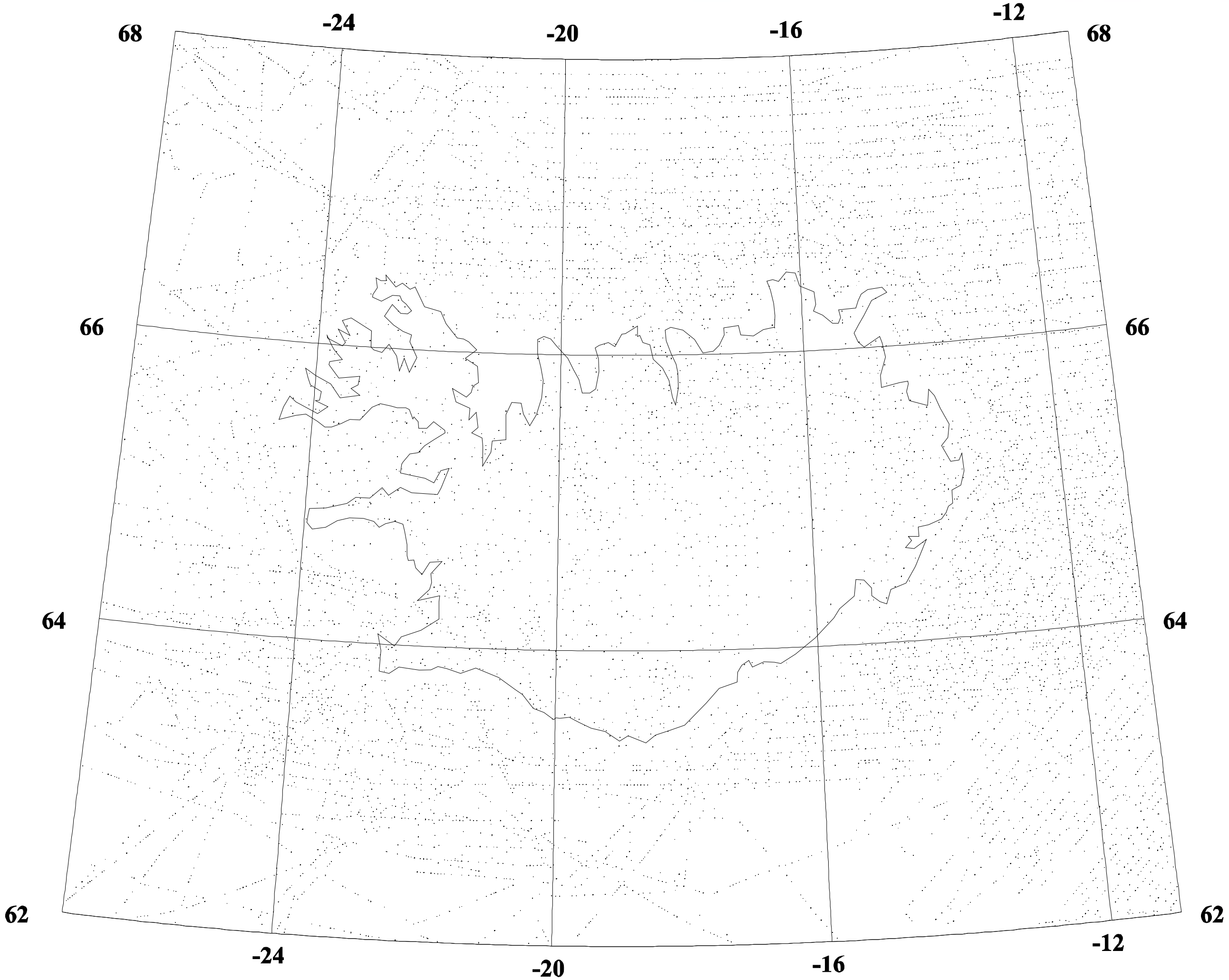
area: $62^\circ N - 68^\circ N$ and $27^\circ W - 11^\circ E$

GOCINA (www.gocina.dk): nat04jpl.geoid90.adj

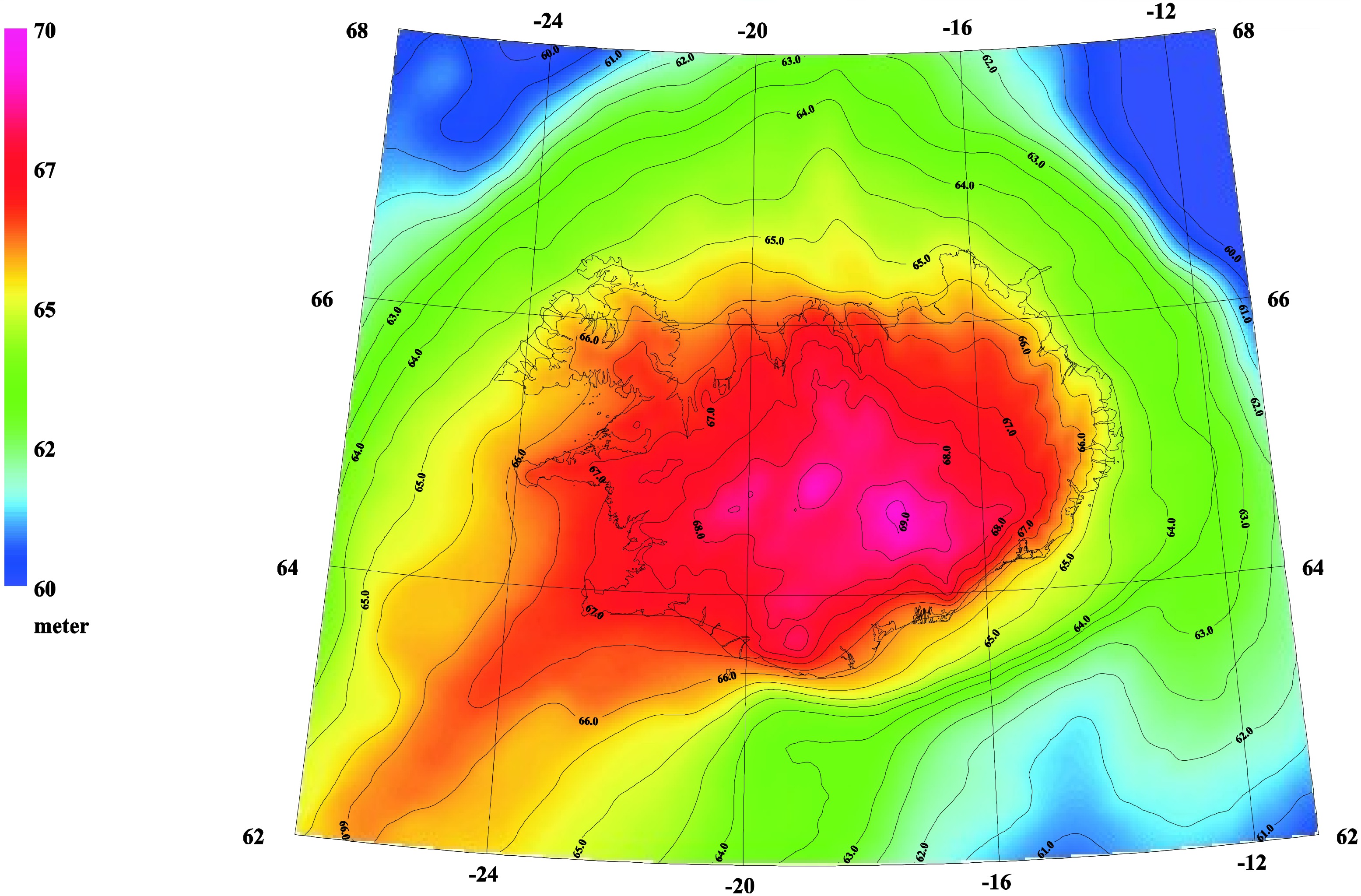
EGM: GRACE1c N=200

spacing: $\Delta\varphi \times \Delta\lambda = 0.05^\circ \times 0.10^\circ$

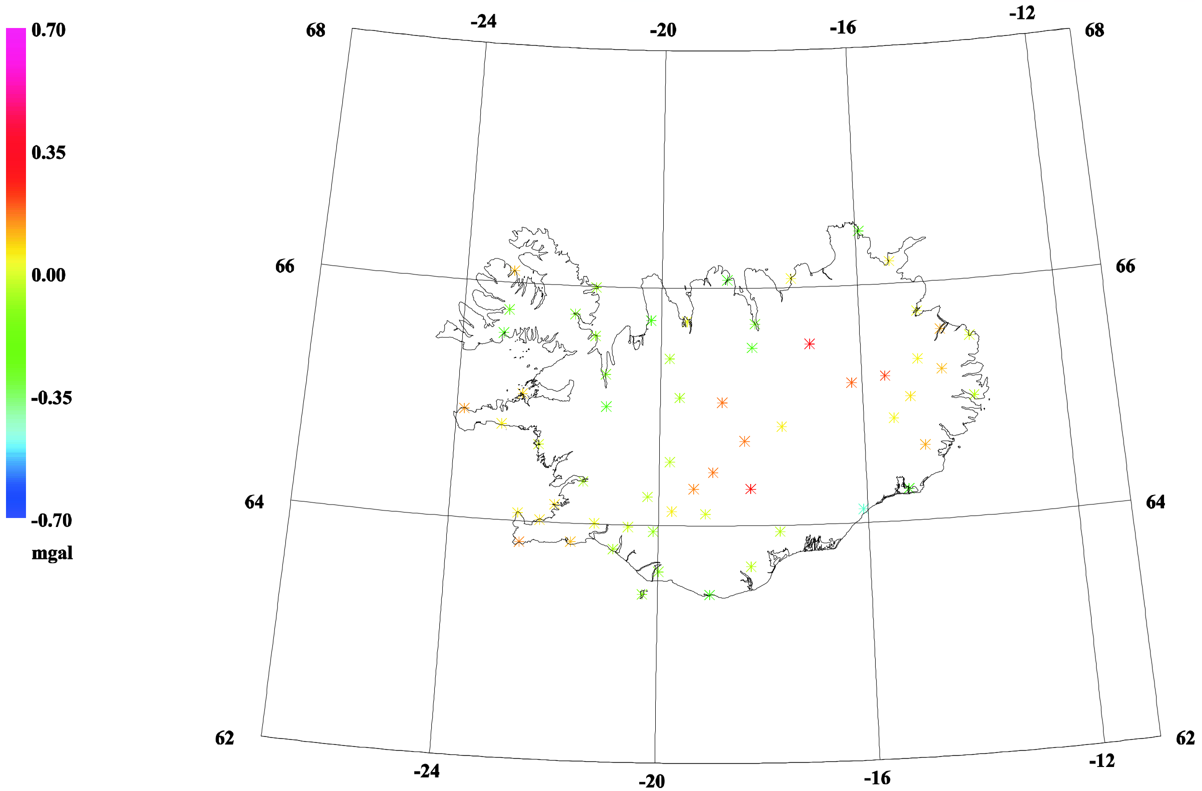
area: $53^\circ N - 84^\circ N$ and $75^\circ W - 60^\circ E$



Iceland. NKG96 geoid. Location of gravity stations.



Iceland. NKG96 geoid (meter)



Iceland. GPS/levelling. levelled geoid heights - NKG96 geoid. (meters)

	mean (m)	std.dev (m)	min. (m)	max. (m)
N_{GPS}-N_{geoid}	-1.272	0.153	-1.659	-0.892
after 4par fit	0.00	0.13	-0.45	0.32

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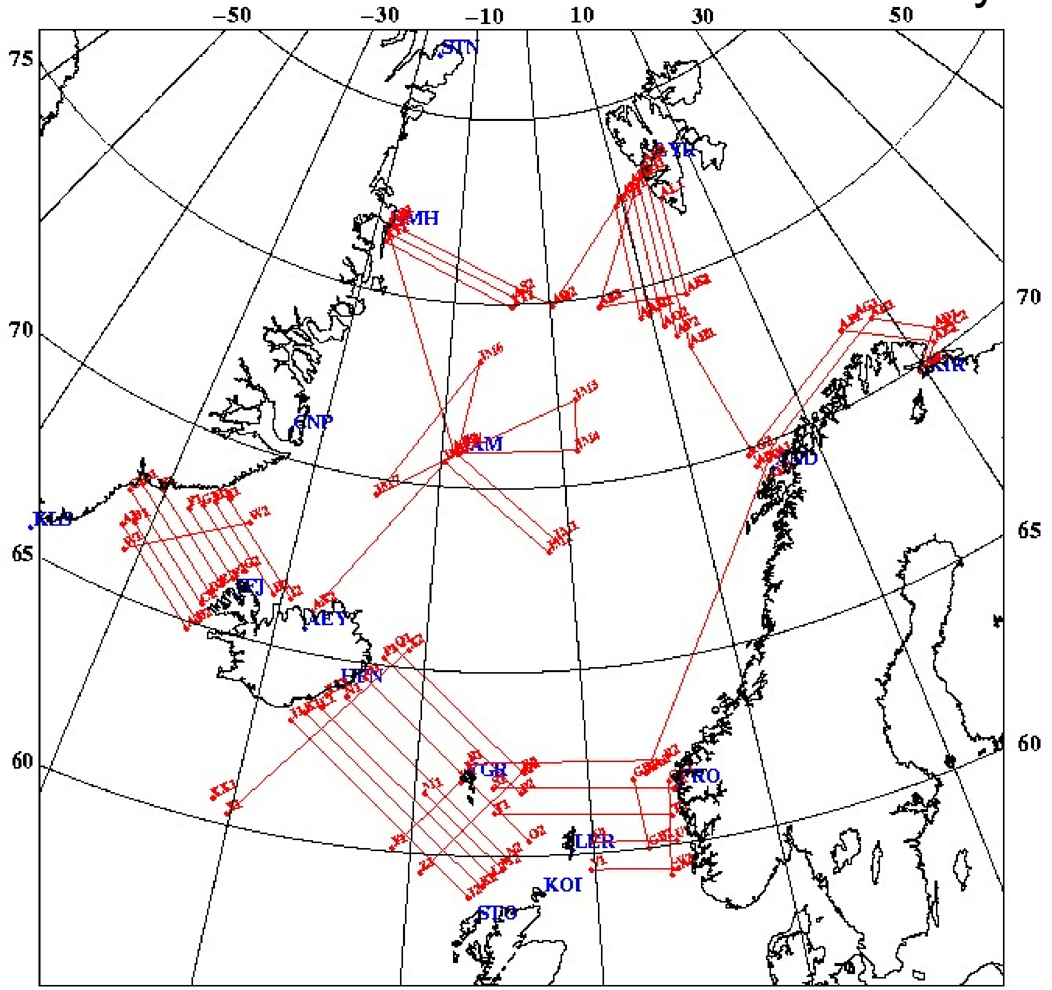
GOCINA/OCTAS 2003 airborne survey



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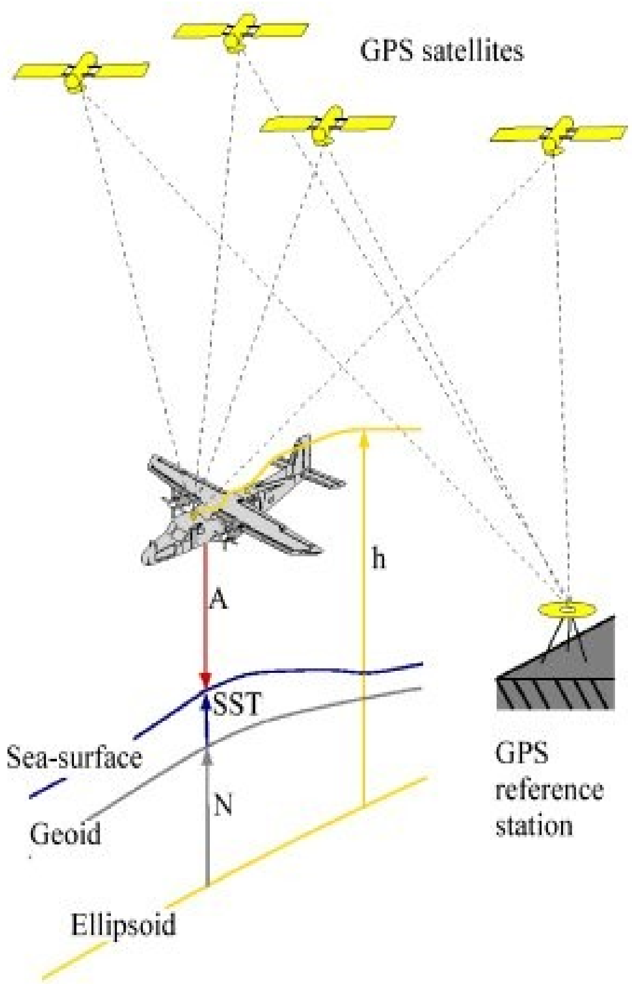

GOCINA airborne results – std.dev < 2 mgal

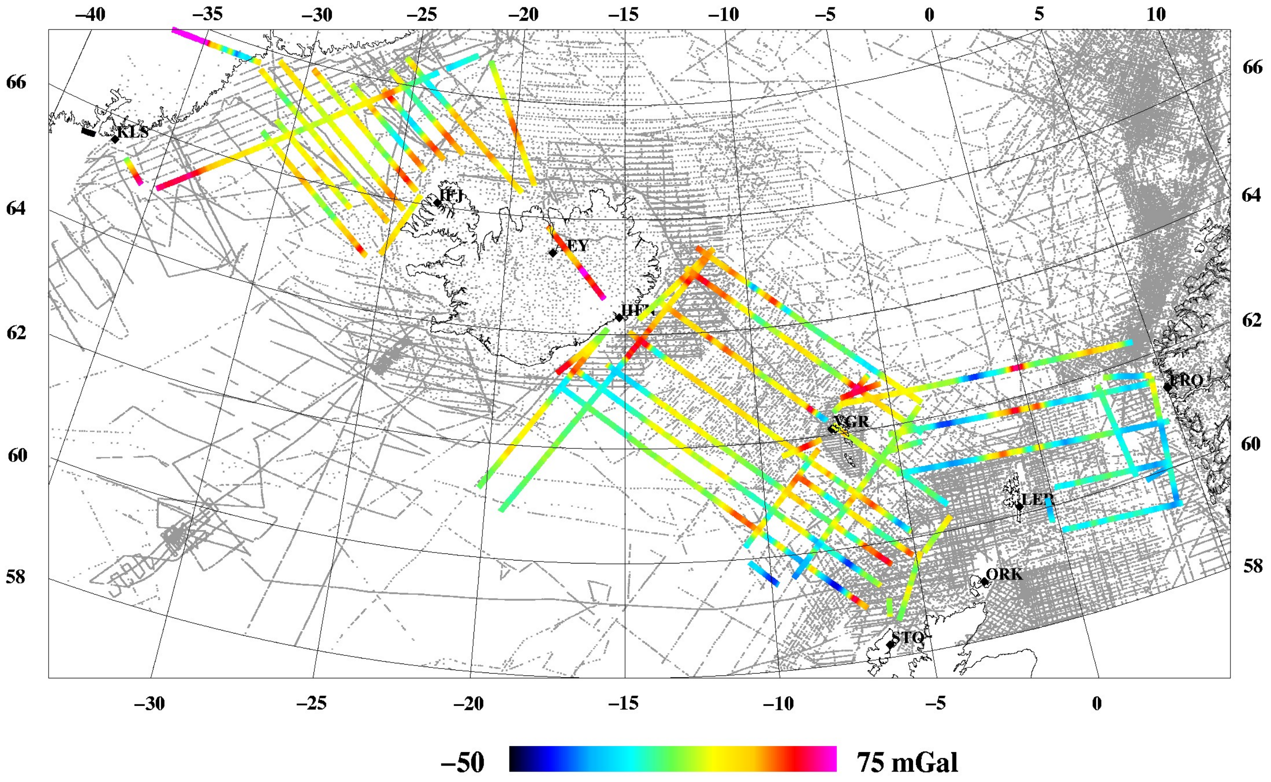

Figure 2. *Airborne free air anomalies. Existing data are shown as gray dots.*

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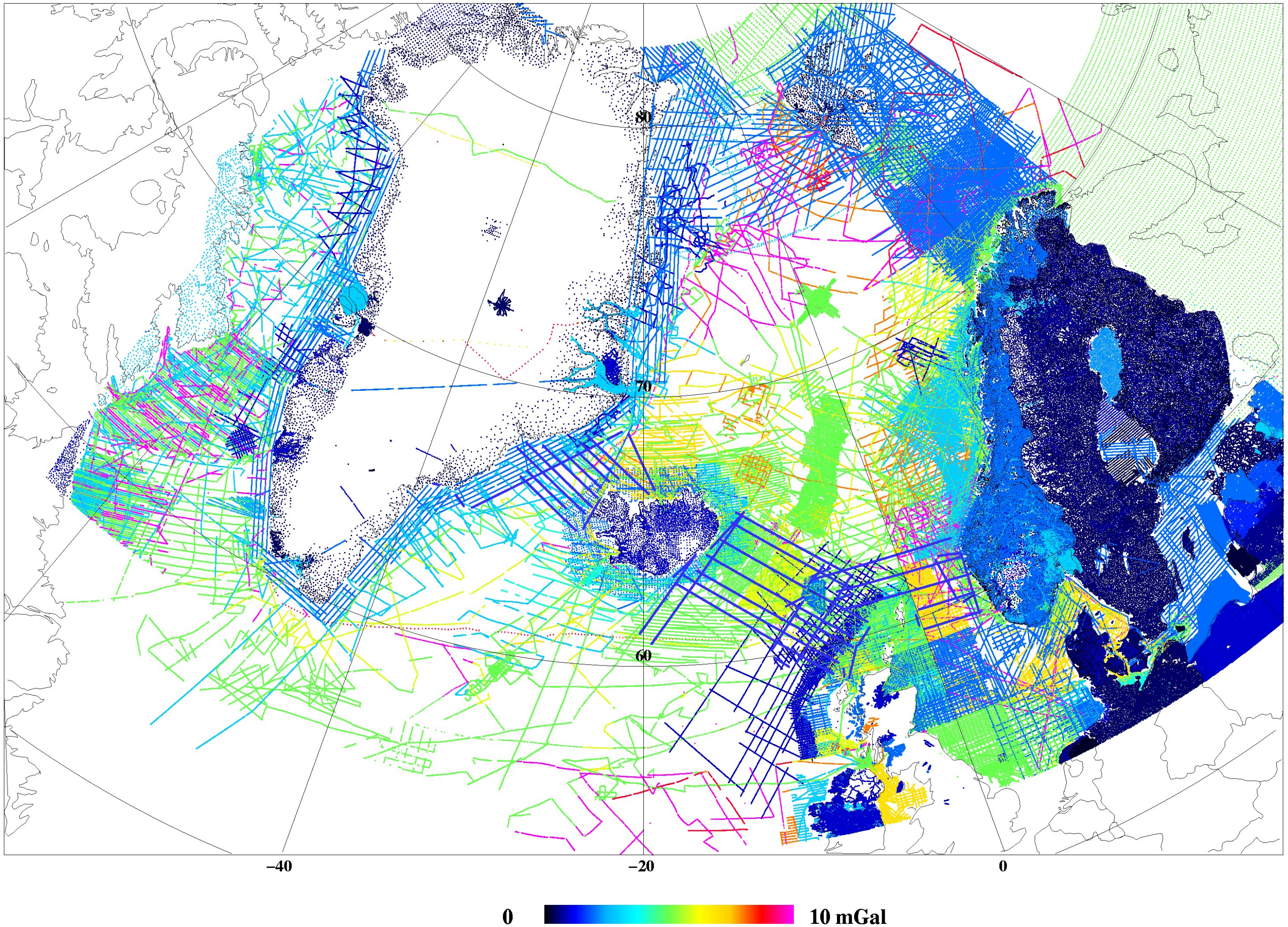
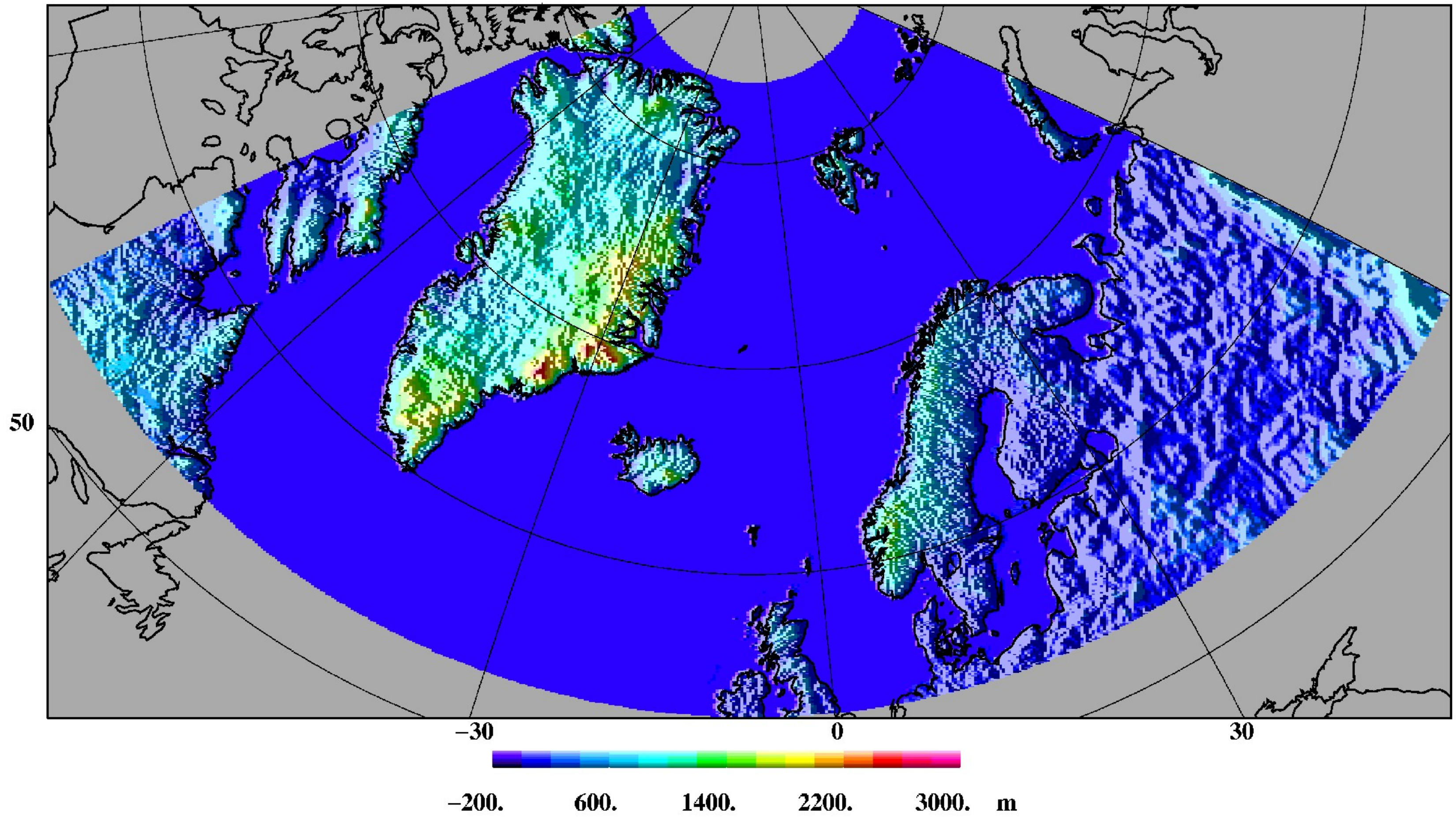
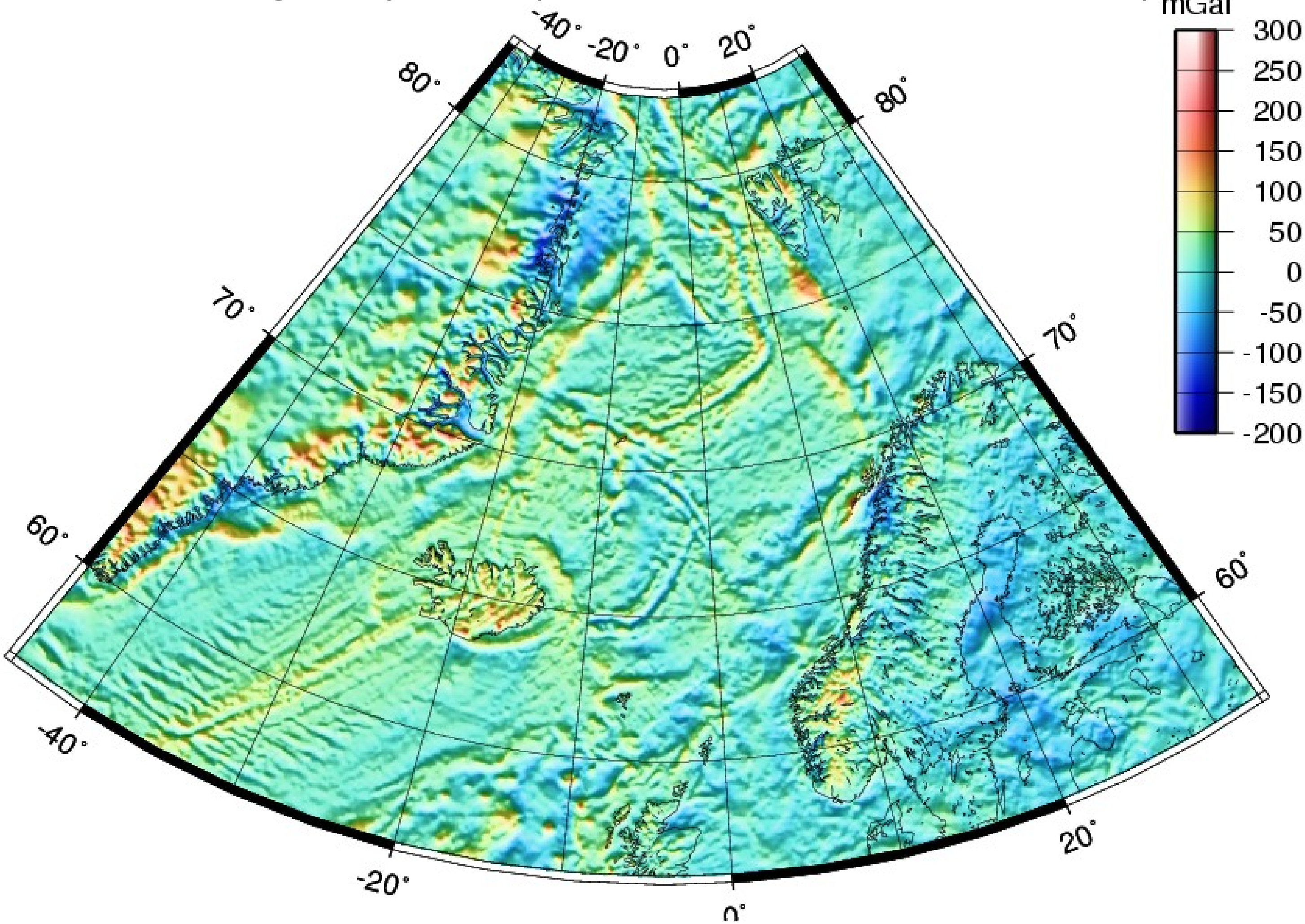
Updated NKG gravity data base (with standard deviations)



Equivalent rock topography



Composite gravity data (free-air at sea/ RTM on land)



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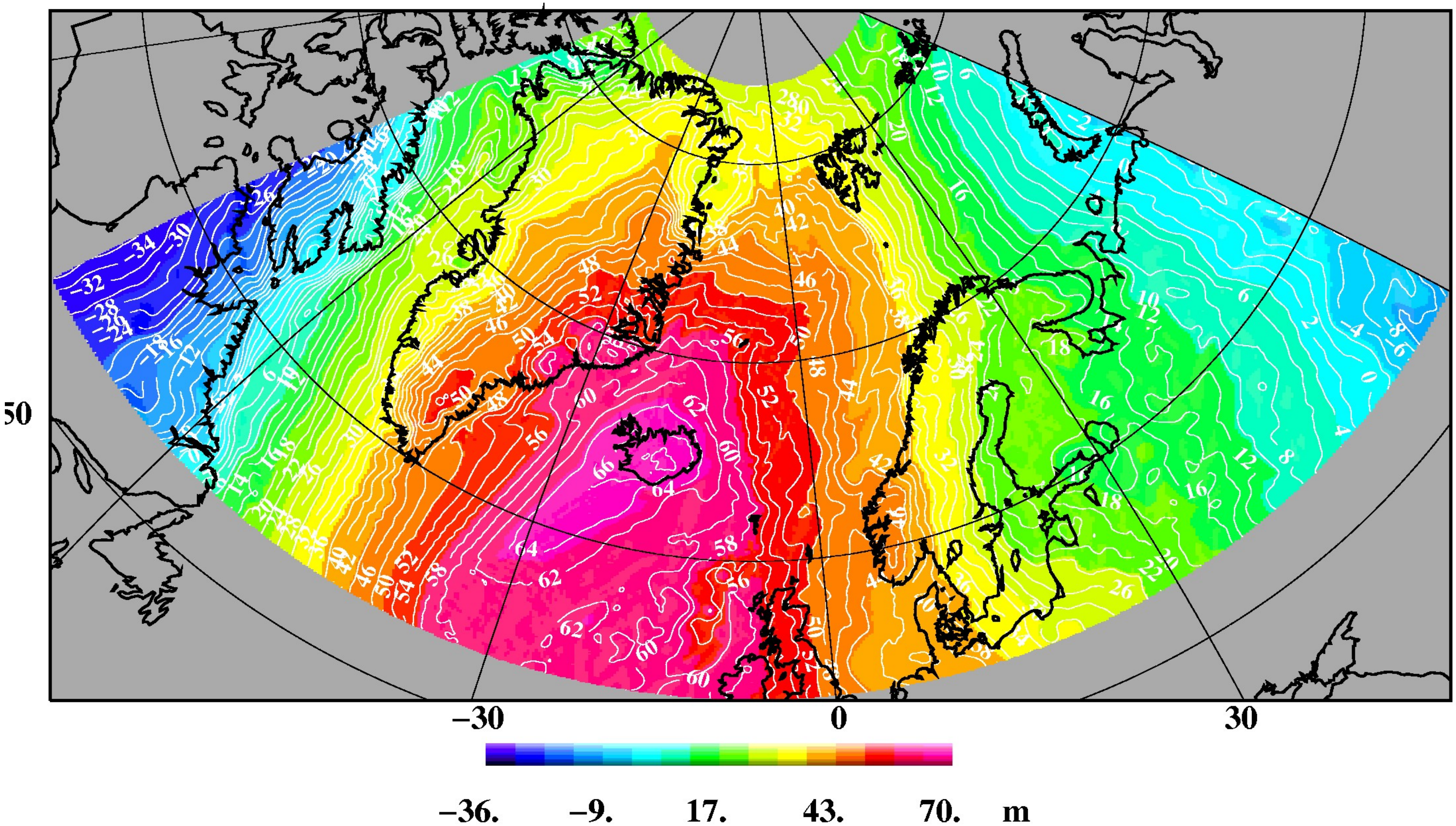
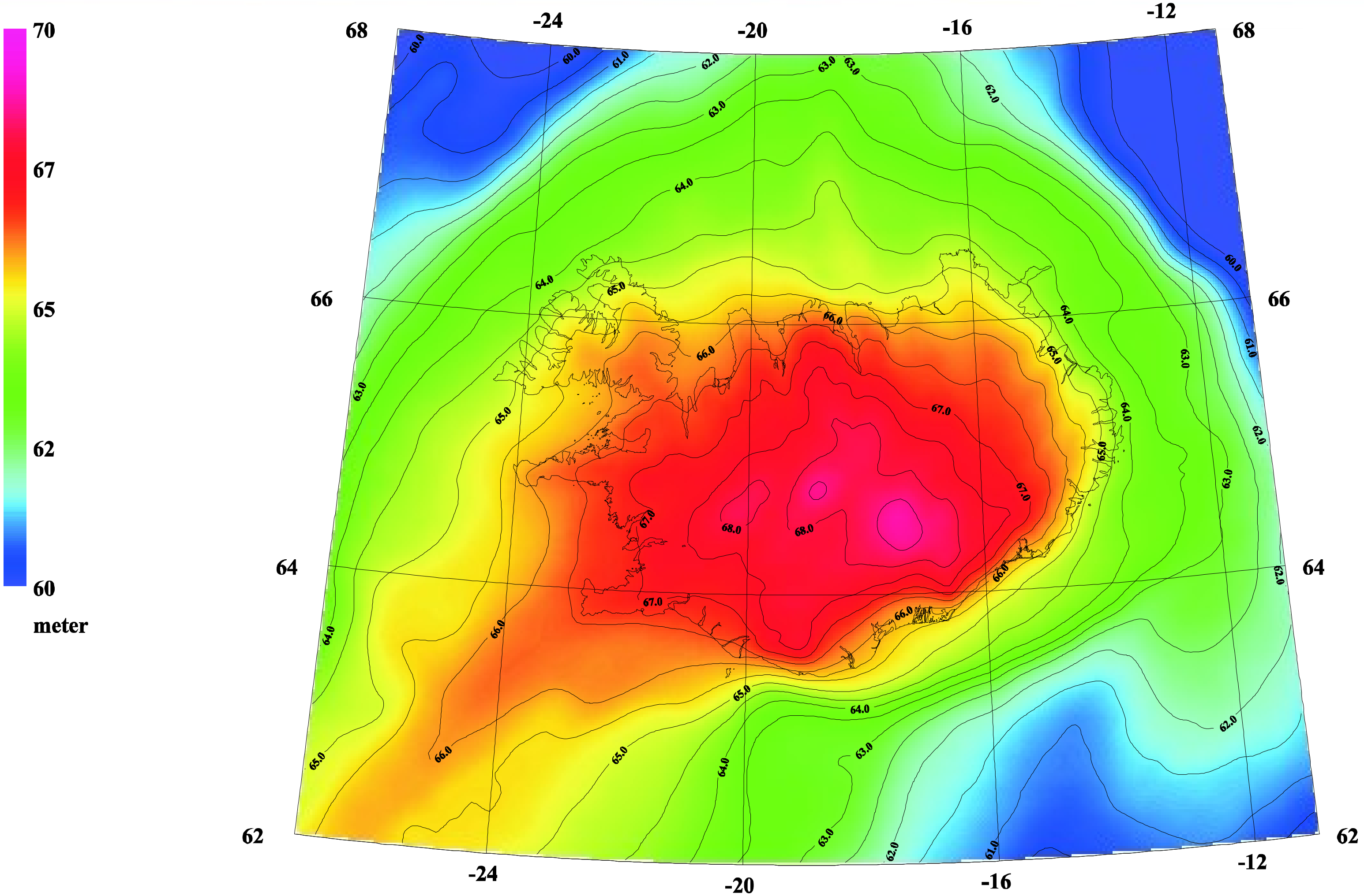
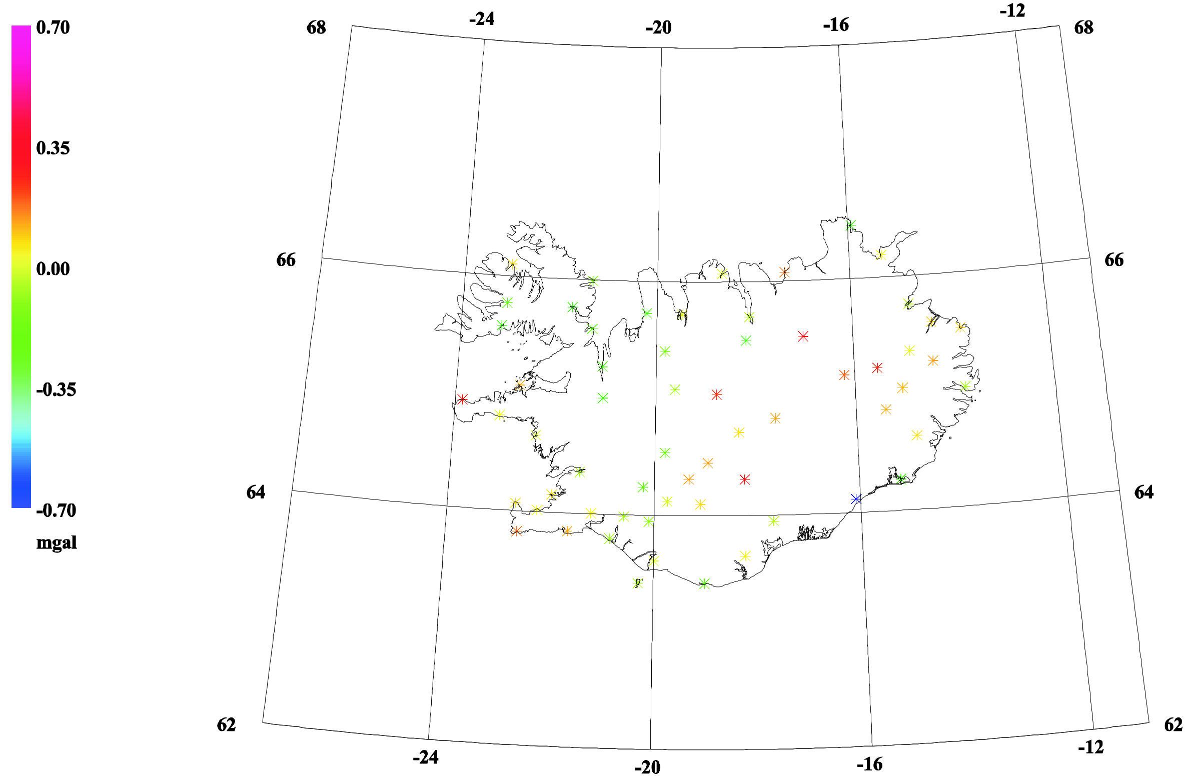
Final gravimetric geoid





Iceland. GOCINA geoid (meter)



Iceland. GPS/levelling. levelled geoid heights - GOCINA geoid. (meters)

	mean (m)	std.dev (m)	min. (m)	max. (m)
N_{GPS}-N_{geoid}	-0.956	0.17	-1.541	-0.575
after 4par fit	0.00	0.15	-0.62	0.30

Conclusions

- Gravimetric geoid.

GOCINA geoid resolves better long wavelengths than NKG96 geoid. However, NKG96 fits better GPS/levelling (better std.dev.). Most probably because of different spacing.

Possible improvement (?): GRACE 2s + EGM96 as EGM
Dense gravity coverage. Updated DEM.

- GPS/levelling: more data – better heights